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NUMERICAL SIMULATION OF THE PROBLEM OF TWO-PHASE FILTRATION IN THE OIL-GAS SYSTEM

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Abstract. The article discusses the issues of numerical simulation of the problem of two-phase filtration in the oil-gas system, and also provides a numerical algorithm for solving the problem and computational experiments on the main indicators of the development of oil and gas fields. The results of the calculation are shown in table two for comparison with different dependences of the relative phase permeability. Numerical results of oil and gas pressure dissipation, as well as oil and gas saturation are presented in visual form in 3D views and contour plots.

Keywords. Filtration, model, algorithm, saturation, permeability, viscosity, porosity, mathematical model, numerical simulation, visual representation.

Introduction

The development of mathematical models, efficient numerical algorithms and software for solving the problems of studying complex oil and gas filtration in heterogeneous reservoirs during the development and production of oil and gas fields is gaining particular popularity. Currently, much attention is paid to the creation of many mathematical models, computational algorithms and software for oil and gas filtration in a porous medium [1]. In high economic performance industries, it is very difficult and expensive to do without mathematical modeling, so we can solve such problems economically by simulating the entire process with mathematical models.

Accordingly, mathematical modeling of such complex processes (in forecasting), numerical study of their solutions and visualization of the situation are currently the most optimal solutions.

Based on this question, we set ourselves the following tasks:

It follows from this that it will be necessary to perform the following tasks: it consists of the formation of information models used in mathematical modeling in the oil and gas system and the development of a calculation algorithm by numerical

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simulation, conducting computational experiments on the main indicators in the oil and gas system and developing a monitoring and forecasting software package.

When implementing these tasks, the main levels of the theory of multiphase filtration are used: the continuity equations, the equation of motion and the equation of state, Darcy's law, numerical modeling methods, as well as modern programming technologies for visualizing numerical results.

To date, a number of scientists have obtained a number of important theoretical and practical results on the issues of filtration of oil and gas systems, including [3-4,6] in their scientific papers considering the modeling of flows containing multicomponent oil mixtures. with complex movements came out and presented a diffusion interface model of a one-component two-phase flow in a porous medium under various conditions. When modeling the filtration processes of a non-stationary two-phase system, the authors used an integral method for solving a system of nonlinear differential equations, performed calculations of the two-phase filtration process and pressure distribution in analytical solutions [5], and applied simplifications. In numerical modeling of filtration processes, the determination of reservoir hydrodynamic parameters based on the multiphase flow model of an oil and gas condensate system, hydrodynamic studies of processes in oil and gas condensate reservoirs, calculation of fluid saturation, pressure and other hydrodynamic parameters were used using finite difference methods and iterative successive approximation, which allows obtaining various distributions [12].

When constructing a mathematical model of the two-phase filtration process, the processes of filtration of a two-phase liquid consisting of oil and water in a non-deformable porous medium [2] were considered without taking into account capillary pressure and gravitational forces. considered in their scientific works the general hydrodynamic problem of capillary pressure without taking into account the considered capillary and gravitational forces and calculated a two-phase flow based on the Buckley-Leverett model [10].

Complex dynamic processes occurring in reservoir conditions with two-sided displacement of oil by gas and water, as well as a model and a numerical algorithm for the problem of three-phase filtration of oil, gas and water in a porous medium are considered [8,14].

In the process of considering some issues of the theory of filtration of porous media [11], an equation was obtained for the pressure of porous media in violation of the Darcy law, and in the numerical analysis of the filtration model of the Dorovsky theory, it was found that the smaller the thickness of the saturated porous layer located between the elastic half-spaces, the more wave amplitude shown by numerical simulation.

Mathematical modeling of multiphase fluid flows in porous media is of great practical importance in oil and gas production.

Filtration of liquids and gases in a porous medium, which occurs during hydrocarbon production, is characterized by the basic laws of conservation of mass, momentum, and energy [7]. However, it is very difficult to directly apply these laws to describe the filtration process in a porous medium; therefore, in practice, instead of the momentum conservation equation, a semi-empirical approach based on the application of Darcy's law is used.

The adequacy of the mathematical model and the determination of the analytical form of the functions of the relative phase permeability coefficient depending on the reservoir parameters, especially the saturation coefficient, play an important role in carrying out computational experiments in two-phase oil-gas filtration [13].

Mathematical model

Consider the boundary value problem of two-dimensional filtration in the oil-gas system, based on the following assumptions. In the reservoir, oil and gas are at a constant temperature and in a state of thermodynamic equilibrium. Gas is mixed with oil, that is, their masses and phases are mixed. Let's assume that gas is dissolved in oil. The layer is assumed to be horizontal, and the effect of gravity is negligible. Then, using the basic equations of the theory of two-phase filtration, we can write the following system of equations describing the non-stationary process of filtration of the oil and gas mixture:

$$\begin{cases} \frac{\partial}{\partial x}\left[\lambda_o\left(\frac{\partial P_o}{\partial x}-\gamma_o\frac{\partial z}{\partial x}\right)\right]+\frac{\partial}{\partial y}\left[\lambda_o\left(\frac{\partial P_o}{\partial y}-\gamma_o\frac{\partial z}{\partial y}\right)\right]= \\ =\frac{\partial}{\partial t}\left[m\rho_o\left(1-S_g\right)\right], \\ \frac{\partial}{\partial x}\left[R_s\lambda_o\left(\frac{\partial P_o}{\partial x}-\gamma_o\frac{\partial z}{\partial x}\right)\right]+\frac{\partial}{\partial y}\left[R_s\lambda_o\left(\frac{\partial P_o}{\partial y}-\gamma_o\frac{\partial z}{\partial y}\right)\right]+ \\ +\frac{\partial}{\partial x}\left[\lambda_g\left(\frac{\partial P_g}{\partial x}-\gamma_g\frac{\partial z}{\partial x}\right)\right]+\frac{\partial}{\partial y}\left[\lambda_g\left(\frac{\partial P_g}{\partial y}-\gamma_g\frac{\partial z}{\partial y}\right)\right]= \\ =\frac{\partial}{\partial t}\left[m\rho_oR_s\left(1-S_g\right)+m\rho_gS_g\right]+\sum_{i=1}^{n}(R_sq_{o_i}+q_{g_i})\delta\left(x-x_i,y-y_i\right), \\ P_{cog}=P_g-P_o, \\ S_o+S_g=1. \end{cases} \quad (1)$$

Here:

$\lambda_l=\frac{K_l}{\mu_l}K\rho_l$ *(l=0, g)* – conductivity *l*- phases;

P_l , ρ_l , γ_l - respectively pressure, density and specific gravity *l* – phases;

δ - Dirac delta function;

K_l - relative permeability for the *l* - phase;

k – absolute permeability;

m – reservoir porosity;

μ_l - viscosity for the l - phase;

ρ_l - the density of the l – phase;

R_s – solubility of oil and gas;

z – the distance from some plane;

q_l – volume of the withdraw flow rate of the l – phase well;

γ_l - the specific weight for the l - phase.

For the convenience of the record we will assume that

For convenience of notation, we assume that

$$\sum_{i=1}^{n} q_{o_i} \delta\left(x - x_i,\, y - y_i\right) = q_o, \quad \sum_{i=1}^{n} q_{g_i} \delta\left(x - x_i,\, y - y_i\right) = q_g .$$

To close the system of equations, the following initial

$$\begin{cases} P_o(x,y,0) = P_o^H(x,y), & P_g(x,y,0) = P_g^H(x,y), \\ S_o(x,y,0) = S_o^H(x,y), & S_g(x,y,0) = S_g^H(x,y) \end{cases} \tag{2}$$

and boundary conditions of the form

$$\frac{\partial P_o}{\partial n}\Big|_{\Gamma} = 0, \qquad \frac{\partial P_g}{\partial n}\Big|_{\Gamma} = 0, \tag{3}$$

or

$$P_o\Big|_{\Gamma} = P_o^H(x,y), \qquad P_g\Big|_{\Gamma} = P_g^H(x,y), \tag{4}$$

where Γ – border of the filtration area;

n – internal border to the boundary of the filtration area.

In this case, condition (3) can be specified on some sections of the boundary, and condition (4) on others.

Solving the boundary value problem (1)-(4) by analytical methods is a difficult task, so we will use the finite difference method. To obtain a finite-difference problem, we use the idea of a variable routing scheme (cross-routing scheme). In this case, it is advisable to write the problem in the form of dimensionless variables.

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To reduce (1)-(4) to a dimensionless problem, we introduce the following notation

$$x = x^* \cdot L,\quad y = y^* \cdot L,\quad P_o = P_o^* \cdot P_H,\quad P_g = P_o^* \cdot P_H,\quad S_o = S_o^* \cdot S_H,\; S_q = S_g^* \cdot S_H,$$

$$\mu_o = \mu_o^* \cdot \mu_H,\; \mu_g = \mu_g^* \cdot \mu_H,\; k = k^* \cdot k_H,\; \tau = t\frac{k_H \cdot P_H}{\mu_H \cdot L^2 \cdot m},\; q^* = Q\frac{\mu_H}{k_H \cdot h \cdot P_H \pi},$$

$$Q = \left(R_S Q_o + Q_g\right).$$

Here

L – characteristic length of the formation;

P_H – some initial characteristic pressure value;

S_H - some initial characteristic saturation value;

h – formation thickness.

In this case, the form of the equations does not change if the former designations for dimensionless functions and variables are retained.

Since the total flow rate is set on the wells, the equations for the pressure in the gas phase must be converted into a total flow.

Let us assume that oil is incompressible, i.e. ρ_o=const, and the gas is compressible, the density of which is expressed in terms of pressure according to the equation of state, i.e. $\frac{p_g}{\rho_g} = RTZ$. Using this ratio and the ratio $P_o = P_g - P_{cog}$, the second equation of system (1) can be represented as:

$$\begin{aligned}
&\frac{\partial}{\partial x}\left[K_o\left(\frac{\partial P_g}{\partial x} - \frac{\partial P_{cog}}{\partial x} - \frac{\gamma_o L}{P_H}\frac{\partial z}{\partial x}\right)\right] + \frac{\partial}{\partial y}\left[K_o\left(\frac{\partial P_g}{\partial y} - \frac{\partial P_{cog}}{\partial y} - \frac{\gamma_o L}{P_H}\frac{\partial z}{\partial y}\right)\right] + \\
&+ \frac{\partial}{\partial x}\left[R_s K_o\left(\frac{\partial P_g}{\partial x} - \frac{\partial P_{cog}}{\partial x} - \frac{\gamma_o L}{P_H}\frac{\partial z}{\partial x}\right)\right] + \frac{\partial}{\partial y}\left[R_s K_o\left(\frac{\partial P_g}{\partial y} - \frac{\partial P_{cog}}{\partial y} - \frac{\gamma_o L}{P_H}\frac{\partial z}{\partial y}\right)\right] + \\
&+ \frac{\mu_o}{\mu_g}\frac{P_H}{\rho_o RTZ}\left\{\frac{\partial}{\partial x}\left[K_g P_g\left(\frac{\partial P_g}{\partial x} - \frac{\gamma_g L}{P_H}\frac{\partial z}{\partial x}\right)\right] + \frac{\partial}{\partial y}\left[K_g P_g\left(\frac{\partial P_g}{\partial y} - \frac{\gamma_g L}{P_H}\frac{\partial z}{\partial y}\right)\right]\right\} = \\
&= \frac{P_H}{\rho_o RTZ}\frac{\partial}{\partial x}\left(P_g S_g\right) + \frac{\partial}{\partial \tau}\left[\left(R_s + 1\right)\left(1 - S_g\right)\right] + \frac{\mu_o L^2}{K\rho_o P_H} q.
\end{aligned} \tag{5}$$

Here $q = \left(R_s Q_o + Q_g\right),\quad \left(q = q^*\right),$

Z - gas compressibility coefficient.

R - inverse gas constant

T – temperaturetemperature.gas compressibility coefficient.

Numerical simulation

To find a numerical solution of eq. (5) with respect to the initial (2) and boundary (3) (or 4) conditions, we use the longitudinal-transverse direction scheme method. This method is the most efficient method for solving second-order parabolic type equations.

We solve equations (5) for the initial (2) and boundary (3) conditions using the method of longitudinal-transverse directions. To obtain a finite difference equation, we use the integro-interpolation method [9]. In the longitudinal direction, we obtain a system of equations for each straight line $y = y_j \ (j = \overline{1, M_y - 1})$ in $t = t_{k+1/2}$:

$$\begin{cases} A_i P_{gi-1,j} - B_i P_{gij} + C_i P_{gi+1,j} = -F_{ij}, & i = \overline{1, M_x - 1}, \\ 4P_{g1j} - 3P_{g0j} - P_{g2j} = 0, \\ 3P_{gM_xj} - 4P_{gM_x-1,j} + P_{gM_x-2,j} = 0. \end{cases} \tag{6}$$

Here

$$P_{gij} = P_g(x_i, y_j, t_{k+1/2});$$

$$A_i = \tilde{K}_{0i-0.5j} + \tilde{R}_{si-0.5j}\tilde{K}_{0i-0.5j} + \frac{\mu_o}{\mu_g}\frac{P_H}{\rho_o RTZ}\tilde{K}_{gi-0.5j}\tilde{P}_{gi-0.5j};$$

$$C_i = \tilde{K}_{0i+0.5j} + \tilde{R}_{si+0.5j}\tilde{K}_{0i+0.5j} + \frac{\mu_o}{\mu_g}\frac{P_H}{\rho_o RTZ}\tilde{K}_{gi+0.5j}\tilde{P}_{gi+0.5j};$$

$$B_i = A_i + C_i + \left(h_1^2/(\tau/2)\right)\frac{P_H}{\rho_o RTZ}\tilde{S}_{gij};$$

$$\begin{aligned} F_i = {} & \frac{h_1^2}{\tau/2}\left\{\frac{P_H}{\rho_o RTZ}\bar{P}_{gij}\bar{S}_{gij} - \left[\left(\bar{R}_{sij}+1\right)\left(1-\tilde{S}_{gij}\right) - \left(\bar{R}_{sij}+1\right)\left(1-\bar{S}_{gij}\right)\right]\right\} - \\ & -\frac{h_1}{h_2}\left\{\bar{K}_{ij+0.5}\left[\bar{P}_{gij+1} - \bar{P}_{gij} - \left(\bar{P}_{cogij+1} - \bar{P}_{cogij}\right) - \gamma_o\frac{L}{P_H}\left(z_{ij+1} - z_{ij}\right)\right] - \right. \\ & -\bar{K}_{ij-0.5}\left[\bar{P}_{gij} - \bar{P}_{gij-1} - \left(\bar{P}_{cogij} - \bar{P}_{cogij-1}\right) - \gamma_o\frac{L}{P_H}\left(z_{ij} - z_{ij-1}\right)\right] + \\ & +\left(\bar{R}_s\bar{K}_o\right)_{ij+0.5}\left[\bar{P}_{gij+1} - \bar{P}_{gij} - \left(\bar{P}_{cogij+1} - \bar{P}_{cogij}\right) - \gamma_o\frac{L}{P_H}\left(z_{ij+1} - z_{ij}\right)\right] - \\ & -\left(\bar{R}_s\bar{K}_o\right)_{ij-0.5}\left[\bar{P}_{gij} - \bar{P}_{gij-1} - \left(\bar{P}_{cogij} - \bar{P}_{cogij-1}\right) - \gamma_o\frac{L}{P_H}\left(z_{ij} - z_{ij-1}\right)\right] + \frac{\mu_o}{\mu_g}\frac{P_H}{\rho_o RTZ}\cdot \\ & \cdot\left\{\left(\bar{K}_{gij+0.5}\bar{P}_{gij+0.5}\right)\left[\bar{P}_{gij+1} - \bar{P}_{gij} - \gamma_g\frac{L}{P_H}\left(z_{ij+1} - z_{ij}\right)\right] - \right. \\ & \left.\left.-\left(\bar{K}_{gij-0.5}\bar{P}_{gij-0.5}\right)\left[\bar{P}_{gij} - \bar{P}_{gij-1} - \gamma_g\frac{L}{P_H}\left(z_{ij} - z_{ij-1}\right)\right]\right\}\right\} - \end{aligned}$$

$$-h_1^2\frac{\mu_o L^2}{K\rho_o P_H}\left(q_o+R_s q_o+q_g\right)-\tilde{K}_{oi+0.5j}\left[\tilde{P}_{cogi+1j}-\tilde{P}_{cogij}-\gamma_o\frac{L}{P_H}\left(Z_{i+1j}-Z_{ij}\right)\right]+$$

$$+\tilde{K}_{oi-0.5j}\left[\tilde{P}_{cogij}-\tilde{P}_{cogi-1j}+\gamma_o\frac{L}{P_H}\left(z_{ij}-z_{i-1j}\right)\right]-$$

$$-\left(\tilde{R}_{si+0.5j}\tilde{K}_{oi+0.5j}\right)\left[\tilde{P}_{cogi+1j}-\tilde{P}_{cogij}+\gamma_o\frac{L}{P_H}\left(z_{i+1j}-z_{ij}\right)\right]+$$

$$+\left(\tilde{R}_{si-0.5j}\tilde{K}_{oi-0.5j}\right)\left[\tilde{P}_{cogij}-\tilde{P}_{cogi-1j}+\gamma_o\frac{L}{P_H}\left(z_{ij}-z_{i-1j}\right)\right]-\frac{\mu_o}{\mu_g}\frac{P_H}{\rho_o RTZ}$$

$$\left\{\left(\tilde{K}_{gi+0.5j}\tilde{P}_{gi+0.5j}\right)\left[\gamma_g\frac{L}{P_H}\left(z_{i+1j}-z_{ij}\right)\right]-\left(\tilde{K}_{gi-0.5j}\tilde{P}_{gi-0.5j}\right)\left[\gamma_g\frac{L}{P_H}\left(z_{ij}-z_{i-1j}\right)\right]\right\},$$

where $\tilde{P},\tilde{S},\tilde{K}$ - approximate values of reservoir pressure, saturation and relative permeability of the reservoir, which are refined during the iteration process.

The system of algebraic equations is tridiagonal. Therefore, its solution can be obtained by the left sweep method on each iterative layer *s*. The criterion for terminating the iterative process for each $\max\limits_{\substack{0\le i\le M_x\\0\le j\le M_y}}\left|P_{gij}^{(s)}-P_{gij}^{(s-1)}\right|\le\varepsilon_p$ j is taken as

Here ε_p - reproducible process accuracy.

P_{gij} in the time layer $t=t_{k+1/2}$ calculated with sufficient accuracy, we $t=t_{k+1}$ go to layer. To do this, we solve the following system of equations on each line:

$$\begin{cases}A_j P_{gi,j-1}-B_j P_{gij}+C_j P_{gi,j+1}=-F_{ij}, & j=\overline{1,M_y-1};\\ 4P_{gi1}-3P_{gi0}-P_{gi2}=0, & \\ 3P_{giM_y}^{(r)}-4P_{giM_y-1}^{(r)}-P_{giM_y-2}^{(r)}=0. & \end{cases}$$

Here

$$P_{gij}=P(x_i,y_j,t_{k+1});$$

$$A_j=\tilde{K}_{0ij-0.5}+\left(\tilde{R}_{sij-0.5}\tilde{K}_{0ij-0.5}\right)+\frac{\mu_o}{\mu_g}\frac{P_H}{\rho_o RTZ}\left(\tilde{K}_{gij-0.5}\tilde{P}_{gij-0.5}\right);$$

$$C_j=\tilde{K}_{0ij+0.5}+\left(\tilde{R}_{sij+0.5}\tilde{K}_{0ij+0.5}\right)+\frac{\mu_o}{\mu_g}\frac{P_H}{\rho_o RTZ}\left(\tilde{K}_g\tilde{P}_g\right)_{ij+0.5};$$

$$B_j = A_j + C_j + \frac{h_2}{\tau/2}\frac{P_H}{\rho_o RTZ}\tilde{S}_{gij};$$

$$\begin{aligned}
F_j = {} & \frac{h_2^2}{\tau/2}\Bigg\{\frac{P_H}{\rho_o RTZ}\left(\bar{P}_{gij}\bar{S}_{gij}\right) - \Big[\left(\tilde{R}_{sij}+1\right)\left(1-\tilde{S}_{gij}\right) - \left(\bar{R}_{sij}+1\right)\left(1-\bar{S}_{gij}\right)\Big]\Bigg\} - \\
& -\frac{h_2}{h_1}\Bigg\{\bar{K}_{oi+0.5j}\left[\bar{P}_{gi+1j} - \bar{P}_{gij} - \left(\bar{P}_{cogi+1j} - \bar{P}_{cogij}\right) - \gamma_o\frac{L}{P_H}\left(z_{i+1j} - z_{ij}\right)\right] - \\
& -\bar{K}_{i-0.5j}\left[\bar{P}_{gij} - \bar{P}_{gi-1j} - \left(\bar{P}_{cogij} - \bar{P}_{cogi-1j}\right) - \gamma_o\frac{L}{P_H}\left(z_{ij} - z_{i-1j}\right)\right] + \\
& +\left(\bar{R}_{si+0.5j}\bar{K}_{oi+0.5j}\right)\left[\bar{P}_{gi+1j} - \bar{P}_{gij} - \left(\bar{P}_{cogi+1j} - \bar{P}_{cogij}\right) - \gamma_o\frac{L}{P_H}\left(z_{i+1j} - z_{ij}\right)\right] - \\
& -\left(\bar{R}_{si-0.5j}\bar{K}_{oi-0.5j}\right)\left[\bar{P}_{gij} - \bar{P}_{gi-1j} - \left(\bar{P}_{cogij} - \bar{P}_{cogi-1j}\right) - \gamma_o\frac{L}{P_H}\left(z_{ij} - z_{i-1j}\right)\right] + \\
& +\frac{\mu_o}{\mu_g}\frac{P_H}{\rho_o RTZ}\Bigg\{\left(\bar{K}_{gi+0.5j}\bar{P}_{gi+0.5j}\right)\left[\bar{P}_{gi+1j} - \bar{P}_{gij} - \gamma_g\frac{L}{P_H}\left(z_{i+1j} - z_{ij}\right)\right] - \\
& -\left(\bar{K}_{gi-0.5j}\bar{P}_{gi-0.5j}\right)\left[\bar{P}_{gij} - \bar{P}_{gi-1j} - \gamma_g\frac{L}{P_H}\left(z_{ij} - z_{i-1j}\right)\right]\Bigg\}\Bigg\} - \\
& -h_2^2\frac{\mu_o L^2}{K\rho_o P_H}\left(q_o + R_s q_o + q_g\right) - \tilde{K}_{oij+0.5}\left[\tilde{P}_{cogij+1} - \tilde{P}_{cogij} + \gamma_o\frac{L}{P_H}\left(z_{ij+1} - z_{ij}\right)\right] + \\
& +\tilde{K}_{oij-0.5}\left[\tilde{P}_{cogij} - \tilde{P}_{cogij-1} + \gamma_o\frac{L}{P_H}\left(z_{ij} - z_{ij-1}\right)\right] - \\
& -\left(\tilde{R}_{sij+0.5}\tilde{K}_{oij+0.5}\right)\left[\tilde{P}_{cogij+1} - \tilde{P}_{cogij} + \gamma_o\frac{L}{P_H}\left(z_{ij+1} - z_{ij}\right)\right] + \\
& +\left(\tilde{R}_{sij-0.5}\tilde{K}_{oij-0.5}\right)\left[\tilde{P}_{cogij} - \tilde{P}_{cogij-1} + \gamma_o\frac{L}{P_H}\left(z_{ij} - z_{ij-1}\right)\right] - \frac{\mu_o}{\mu_g}\frac{P_H}{\rho_o RTZ} \\
& \Bigg\{\left(\tilde{K}_{gij+0.5}\tilde{P}_{gij+0.5}\right)\left[\gamma_g\frac{L}{P_H}\left(z_{ij+1} - z_{ij}\right)\right] - \left(\tilde{K}_{gij-0.5}\tilde{P}_{gij-0.5}\right)\left[\gamma_g\frac{L}{P_H}\left(z_{ij} - z_{ij-1}\right)\right]\Bigg\}
\end{aligned}$$

Here

$\tilde{P},\tilde{S},\tilde{K}$ - iterative values of reservoir pressure, saturation and relative permeability of the reservoir, respectively, which are determined during the iteration process. $\tilde{P}_{ij\ k+1}^{(s)}$ The initial value is taken from the k+0.5th time layer, i.e.

$\tilde{P}_{ij\ k+1}^{(0)} = P_{ijk+0,5}$, $\tilde{K} = K(\tilde{S})$;

$\overline{P},\overline{S},\overline{K}$ - *k+0,5* - m is the pressure value in the temporary reservoir, saturation and relative permeability of the reservoir.

The iterative process is tested based on the following condition.

$$\max_{\substack{0\le i\le M_x \\ 0\le j\le M_y}} \left| P_{gij}^{(s)} - P_{gij}^{(s-1)} \right| \le \varepsilon_p$$

here ε_p - the accuracy of the iterative process.

The share of well flow rates in the oil phase is determined as follows:

$$q_o = \frac{KK_o}{\mu_o}\left\{\frac{\partial P_o}{\partial n} - \gamma_o \frac{\partial z}{\partial n}\right\}$$

$\tilde{S}_{oij}$ to calculate saturation, we replace the first equation of the system of equations (1) with finite-difference analogs corresponding to differential operators. After simple substitutions, we get the following formula

$$\tilde{S}_{oij} = \overline{S}_{oij} + \frac{\tau/2}{h^2}\left\{K_{oi+1/2}\left(P_{oi+1j} - P_{oij} - \gamma_o \frac{L}{P_H}\left(z_{i+1j} - z_{ij}\right)\right) - \right.$$

$$\left. -K_{oi-1/2}\left[P_{oij} - P_{i-1j} - \gamma_o \frac{L}{P_H}\left(z_{ij} - z_{i-1j}\right)\right]\right\} + \frac{\tau/2}{h^2}\cdot$$

$$\cdot\left\{K_{oij+0.5}\left(P_{oij+1} - P_{oij} - \gamma_o \frac{L}{P_H}\left(z_{ij+1} - z_{ij}\right)\right) - \right.$$

$$\left. -K_{oij-0.5}\left[P_{oij} - P_{oij-1} - \gamma_o \frac{L}{P_H}\left(z_{ij} - z_{ij-1}\right)\right]\right\} - \tau\frac{\mu_o L^2}{K\rho_o P_H}\cdot q_o,$$

where the values of K and P are taken on k + 1 time layer, which is refined, and the values $\overline{S}_{oij}$ are taken at the k-th time layer.

The criterion for terminating the iterative process with respect to the parameter *m* is the relation

$$\max_{\substack{0\le i\le M_x \\ 0\le j\le M_y}} \left| S_{oij}^{(s)} - S_{oij}^{(s\text{-}1)} \right| \le \varepsilon_s$$

where ε_s - the accuracy of the iterative process.

After calculation S_{oij} with sufficient accuracy we calculate the values of gas saturation S_{gij} and pressure in the oil phase P_{oij} on each time layer according to the following formulas:

$$S_{gij} = 1 - S_{oij},$$
$$P_{oij} = P_{cogij} - P_{gij}.$$

Algorithm for solving the boundary value problem.

On the basis of the above calculation scheme, an algorithm for solving the problem has been developed. The algorithm included a rather complicated process. It is mainly based on the finite difference method and the iterative method, and is implemented as follows.

1. Specify the original data values
- number of iterations in time – *nt*;
- permeability value –*k*;
- porosity value –*m*;
- layer length –*L*;
- viscosity coefficient value газа – μ ;
- reservoir gas pressure –*P*;
- gas and oil saturation factor – S_g, S_o;

2. Repeating a cycle in time *k=1…nt;*

3. **First stage.** This step consists of performing calculations in the time layer *k+0,5*. In this case *(j=1...n-1)* for each value of j, the following is true:

3.1. Coefficients of finite-difference equations are calculated *(i=1..n-1);*

3.2. A_0; B_0 drive coefficients are determined from the left boundary conditions;

3.3. A_i; B_i *(i=1,n-1)* race odds are calculated from left to right up to ten;

3.4. $P_{g\,n,j}$ is determined from the right boundary condition;

3.5. $P_{g\,i,j}$ *k+0.5* – calculated in the time layer;

3.6. B this time layer, the iterative process is checked

$$\left| P^{(s)}_{g\ i,j} - P^{(s-1)}_{g\ i,j} \right| \le \varepsilon_p \quad .$$

If this iteration condition is met, then go to the next step 3.7, otherwise return to 3.1.

Here

$P^{(s)}_g, P^{(s-1)}_g$ - two values next to each other P_g pressure functions *(s* is the number of iterations, $s-1$ previously calculated value in , its zero value is taken from the initial value, if there are subsequent values, then it will be the previous value);

- ε_p - iteration precision.

3.7. P_g - the gas pressure in the reservoir is calculated with sufficient accuracy;

3.8. P_o - the oil pressure in the reservoir is calculated by the following formula:

$$P_o = P_g - P_{cog}.$$

3.9. Saturation by approximation of the second equation of the system of equations S_o We calculate the coefficient using the following formula

$$S_o = \hat{S}_o + \frac{\Delta\tau}{\Delta x^2}\frac{\mu_g}{\mu_o}\left(K_{oi-0.5,j}\hat{P}_{o\,i-1,j} - (K_{o\,i-0.5,j} + K_{o\,i+0.5,j})\hat{P}_{o\,i,j} + K_{oi+0.5,j}\hat{P}_{o\,i+1,j}\right).$$

3.10. Since the system of equations is non-linear with respect to saturation functions, an iterative process is used. The iterative process continues until the following condition is met

$$\left|S^{(s)}_{o\,i,j} - S^{(s-1)}_{o\,i,j}\right| \le \varepsilon_s \,.$$

Here

$S^{(s)}_o, S^{(s-1)}_o$ - two values next to each other S_o pressure functions (s – present value, $s - 1$ one previously calculated value);

ε_s - iteration precision.

3.11. S_o Oil saturation in the first and second layers is determined with sufficient accuracy by the following formula: $S_g = 1 - S_n$.

3.12. If the iterative process is completed, it proceeds to the next stage, otherwise it returns to paragraph 3.1;

4. **Second stage.** At the second stage, the above calculations $k+1$ the same is done in the time layer, that is, from point 3.1 to point 3.12.

5. Found $k+1$ the solutions in the layer will be the starting point for the next $k+1$ step.

6. Numerical results are displayed in tabular and 3D graphic form.

7. End of iteration over time. If the specified number of iterations coincides with the iteration cycle time, the program stops, otherwise it returns to point 2.

The developed algorithm is easily integrated for two- and three-difference equations, and can also be applied to other multiphase filtration problems, such as oil-gas and oil-gas-water systems.

Computational experiments.

An efficient algorithm for the numerical solution of problems (1) - (5), implemented on a computer, has been developed. In order to study the suitability of the algorithm for calculating the pressure and saturation fields, we considered two dependences of these functions.

In the first case, they are taken from in the form

$$K_g(S_g) = \begin{cases} 0, & 0 \le S_g \le 0{,}1; \\ \left(\dfrac{S_g - 0{,}1}{0{,}9}\right)^{3{,}5}\left[1 + 3\left(1 - S_g\right)\right], & 0{,}1 \le S \le 1. \end{cases} \tag{8}$$

$$K_o(S_g)=\begin{cases}\left(\dfrac{0,8-S_g}{0,8}\right)^{3.5}, & 0\le S_g\le 0,8;\\ 0, & 0,8\le S_g\le 1.\end{cases}$$

In the second case, it is determined by the results of experiments conducted at UzbekNIPIneftegaz, according to the formulas

$$K_g(S_g)=\begin{cases}0, & 0\le S_g\le 0,4\\ 8,59667S_g^3-9,16878S_g^2+3,3075S_g-0,362522, & S_g>0,4\end{cases}$$

(9)

$$K_o(S_g)=\begin{cases}0, & 0,8\le S_g\le 0,1;\\ -6,16392S_g^3-9,16207S_g^2-5,16941S_g+1,00871, & S_g>0,8\end{cases}$$

The solubility function of gas in oil has the form

$$R_s=11,3+0,75P_{o.}$$

To conduct numerical studies and determine the suitability of the model, we consider a circular reservoir with a diameter of 10 km, which is developed by a single central well with a constant flow rate.

q=500000м^3/сут. K=0,2Д, m=0,1, P^H=300ат, S_g^H=0,8; S_o^H=0,2; N=21, M=21, μ_o=3 СПз, μ_g=0,01 СПз.

To estimate the error of the results obtained, the material balance equation is used, although it itself is the average integral solution of the problem and has a certain error.

In table. Figure 1 shows the changes in time and the average reservoir pressure, calculated from the solutions of finite difference equations and the material balance. For clarity, relative values are also given.

The errors of these two quantities. Table 1 shows that for 720 days the relative errors of these values do not exceed 1%, which shows their closeness.

Table 1

Comparison of the average reservoir pressure by two methods

Development time (days)	Gas pressure in the well, (atm)	Average formation pressure, (atm)		Relative error, %
		EHM	material balance	
40	284,82	299,04	299,76	0,0380
120	281,79	298,92	299,28	0,1230
240	279,66	298,75	298,56	0,2776
480	275,33	295,32	297,12	0,6060
720	273,03	292,92	295,68	0,9387

To analyze the convergence of the iterative process of solving nonlinear algebraic dependences of the results obtained with $\varepsilon=0,001$, $\varepsilon=0,0001$ and

$\varepsilon = 0,00001$. An analysis of the results presented in Table 3 shows that with an increase in the iteration accuracy, the pressure value does not change sharply. Decreasing value ε the value of pressure changes at all points of the filtration area, i.e. at low precisions, the results will be higher. However, they are small so that it can be taken with sufficient accuracy in the calculations $\varepsilon = 0,001$.

Table 2 compares the results of calculations performed using formulas (8) and (9), which shows that the calculated pressure values are close.

Table 2

Change in well pressure and formation pressure with different dependencies of relative phase permeabilities

Days	According to literature		According to UzbekNIPIneftgaz	
	(8) in accordance with the formula		(9) in accordance with the formula	
	$P_{скв}$	$P_{ср}$	$P_{скв}$	$P_{ср}$
40	0,9490	0,9988	0,9494	0,9988
120	0,9385	0,9964	0,9393	0,9964
240	0,9314	0,9925	0,9322	0,9925
480	0,9203	0,9846	0,9211	0,9844
720	0,9095	0,9766	0,9101	0,9764

Figures 1 and 2 show 3D and contour plots of oil and gas pressure and saturation factors during the operation of five wells symmetrically located in the center of the formation. Figure 1 shows that the pressure around the well decreases to 0.85, the oil saturation coefficient decreases to 0.193, and the gas saturation coefficient increases to 0.807. These are indicators that when the pressure near the well drops, gas saturation increases, and oil saturation decreases (Fig. 2).

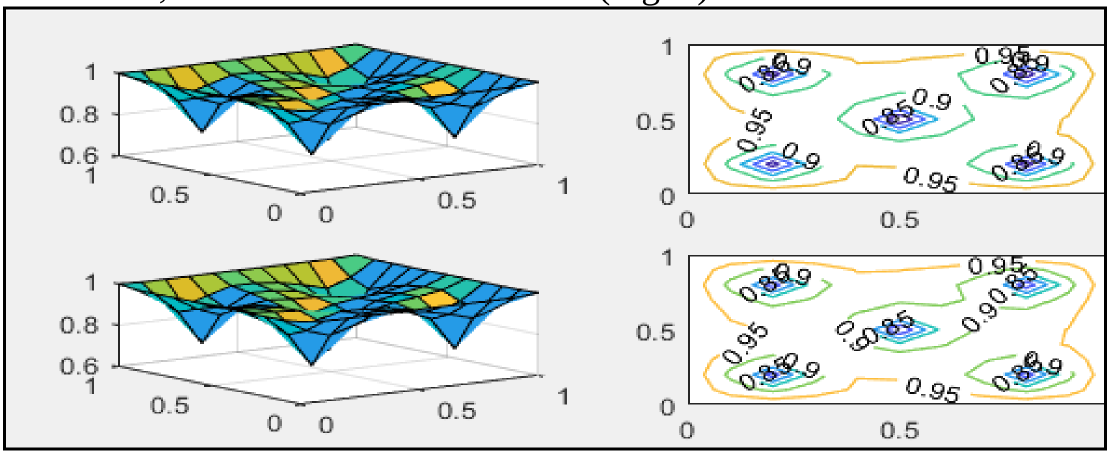

Fig.1. 3D and contour plots of oil and gas pressure distribution in the reservoir

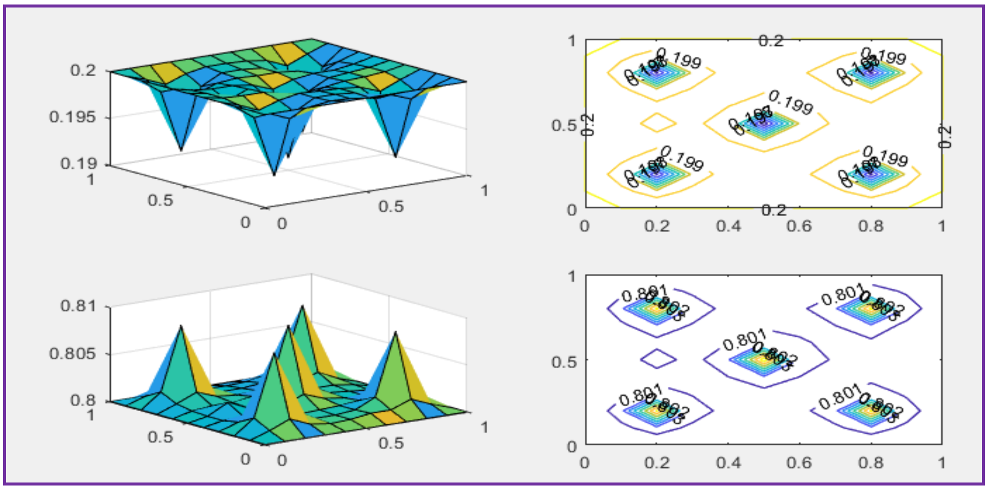

Figure 2. 3D state and contour plots of reservoir oil and gas saturation values

Conclusion.

An analysis of the results shows that the numerical algorithm proposed by us is suitable for calculating pressure and saturation fields for various boundary and initial data, and the mathematical model adequately describes the process under consideration. They can be used to calculate the main indicators in the design and analysis of the development of oil and gas fields.

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