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NUMERICAL SIMULATION OF THE PROBLEM OF GAS FILTRATION IN A PIECEWISE INHOMOGENEOUS POROUS MEDIUM

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Abstract: It is known that one of the main indicators of the development of gas fields are changes in reservoir and bottom hole pressure.

These indicators can be determined using mathematical modeling of unsteady gas filtration in a porous medium under appropriate boundary conditions that adequately describe the process as a whole. Due to the complexity and nonlinearity of the two-dimensional differential equations of gas filtration, it is currently not possible to obtain the necessary analytical solutions. Therefore, to calculate the main indicators of the development of gas fields, various numerical methods, computational algorithms and their software tools for conducting SE on a computer were proposed.

Keywords: Mathematical model, filtration, algorithm, slow permeable layer, porous medium, iterative method, quasi-linear method, finite difference method, software, gas.

Introduction. The use of modern numerical methods for solving the problem of gas filtration in a piecewise inhomogeneous porous medium makes it possible to conduct research on the main indicators of gas field development using a computational experiment.

When solving the issues of design, management, forecasting and analysis of the development of gas and gas condensate fields, the use of mathematical modeling

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methods, as well as modern computer technologies, simplifies the solution of the task. This makes it possible to accelerate the design and analysis of the development of gas and gas condensate fields.

Mathematical model. It is known that the heterogeneity of the layer significantly affects the filtration flows in a porous medium. In general, the movement of fluid in a piecewise inhomogeneous porous layer depends on the following formation parameters:

- permeability coefficient;
- coefficient of porosity;
- layer thickness factor.

Taking into account these characteristics of the layers, the mathematical model of the process of unsteady gas filtration in a piecewise inhomogeneous porous medium is described by nonlinear differential equations of parabolic type, and is written as the following boundary value problem [1,7,10]:

$$2amh\frac{\partial P}{\partial t}=\frac{\partial}{\partial x}\left(\frac{kh}{\mu}\frac{\partial P^2}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{kh}{\mu}\frac{\partial P^2}{\partial y}\right)-Q,\ \ (x,y)\in G \tag{1}$$

$$P(x,y)=P_H(x,y),\ \ t=0,\ \ (x,y)\in G \tag{2}$$

$$-\frac{kh}{\mu}\frac{\partial P}{\partial n}=\alpha\left(P_A-P\right),\ \ (x,y)\in\Gamma \tag{3}$$

$$\oint_{s_{i_q}}\frac{kh}{\mu}\frac{P}{P_{am}}\frac{\partial P}{\partial n}ds=-q_{i_q}(t),\ \ (x,y)\in s_{i_q},\ i_q=1,2... \tag{4}$$

$$k(x,y)=\begin{cases}k_1, & (x,y)\in G_{k1},\\ k_2, & (x,y)\in G_{k2},\\ \dots & \dots\ \ \dots.\\ k_n, & (x,y)\in G_{kn}.\end{cases}\qquad h(x,y)=\begin{cases}h_1, & (x,y)\in G_{h1},\\ h_2, & (x,y)\in G_{h2},\\ \dots & \dots\ \ \dots.\\ h_n, & (x,y)\in G_{hn}.\end{cases}$$

Here

P - layer pressure;

P_H - initial layer pressure;

P_A - border pressure;

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P_{am} - atmospheric pressure;

μ - dynamic viscosity of the gas;

k - layer permeability coefficient;

h - layer thickness;

q_{i_q} - debit i_q - th well;

N_q - number of wells;

m - layer porosity coefficient;

δ - Dirac function;

a - gas saturation coefficient;

$$\alpha = \begin{cases} 0, & \text{–fixed at the boundary;} \\ 1, & \text{–not fixed at the boundary.} \end{cases}$$

For a numerical solution of problem (1)-(4), we pass to dimensionless variables using the following formulas:

$$P^* = P / P_H; \quad x^* = x / L; \quad y^* = y / L; \quad k^* = k / k_0; \quad h^* = h / h_0;$$

$$\tau = \frac{k_0 P_0 t}{am\mu L^2}; \quad q^* = \frac{P_{am} q \mu}{\pi k_0 P_0^2 h_0}, \quad \alpha^* = \frac{\mu L}{k_0 h_0}.$$

Here P_o -characteristic value of pressure in the area of gas content; k_o - characteristic value of layer permeability; h_o - characteristic value of the layer thickness; L - characteristic length.

Then problem (1)-(4) is written in dimensionless form. Skipping the "*", we write it in the following form:

$$h\frac{\partial P}{\partial \tau} = \frac{\partial}{\partial x}\left(kh\frac{\partial P^2}{\partial x}\right) + \frac{\partial}{\partial y}\left(kh\frac{\partial P^2}{\partial y}\right) - q, \quad (x,y)\in G \tag{5}$$

$$P(x,y,t_0) = P_H(x,y), \quad t=0, \quad (x,y)\in G \tag{6}$$

$$-kh\frac{\partial P(x,y,t)}{\partial n} = \alpha(P_A - P), \quad (x,y)\in \Gamma \tag{7}$$

$$\oint_{s_{i_q}} khP\frac{\partial P}{\partial n}ds = -q_{i_q}(t), \quad (x,y)\in s_{i_q} \quad i_q = 1,2... \tag{8}$$

To solve problem (5)-(8), we use the finite difference method. At the same time, to obtain a difference problem, the algorithmic idea of an implicit scheme of alternating directions and the sweep method in each direction are used [2,7,11,12].

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Numerical model. To pass from the differential problem to the difference region of the filtration $G + \Gamma$, we put in correspondence the grid region constructed as follows:

$$\Omega_{xyt} = \left\{ \left(x_i = i\Delta x,\ y_j = j\Delta y,\ t_l = l\,\tau \right);\ i = \overline{1, N};\ j = \overline{1, M},\ l = \overline{0, N_\tau},\ \tau = \frac{1}{N_\tau} \right\},$$

where N - number of nodes on the line y_j; M - number of nodes on the line x_i; $\Delta x,\ \Delta y$ - grid steps.

Then the scheme of alternating directions in the variable x, used for the partial differential equation (5) and describing the filtering process, in the area of equations can be replaced by the following difference relation for the $\tau + 0.5$ nd time layer [10]:

$$\frac{\overline{P}_{i,j} - \hat{P}_{i,j}}{\tau / 2} = \frac{T_{i-0.5,j}\overline{P}^2_{i-1,j} - (T_{i-0.5,j} + T_{i+0.5,j})\overline{P}^2_{i,j} + T_{i+0.5,j}\overline{P}^2_{i+1,j}}{(\Delta x)^2} + \\ + \frac{T_{i,j-0.5}\hat{P}^2_{i,j-1} - (T_{i,j-0.5} + T_{i,j+0.5})\hat{P}^2_{i,j} + T_{i,j+0.5}\hat{P}^2_{i,j+1}}{(\Delta y)^2} - \delta_{i,j} q_{i,j}; \qquad (9)$$

$$\frac{P_{i,j} - \overline{P}_{i,j}}{\tau / 2} = \frac{T_{i-0.5,j}\overline{P}^2_{i-1,j} - (T_{i-0.5,j} + T_{i+0.5,j})\overline{P}^2_{i,j} + T_{i+0.5,j}\overline{P}^2_{i+1,j}}{(\Delta x)^2} + \\ + \frac{T_{i,j-0.5}P^2_{i,j-1} - (T_{i,j-0.5} + T_{i,j+0.5})P^2_{i,j} + T_{i,j+0.5}P^2_{i,j+1}}{(\Delta y)^2} - \delta_{i,j} q_{i,j}. \qquad (10)$$

Here $P_{i,\mathrm{j}}$ - value of the gas pressure at the *l+1*-th time layer; $\overline{P}_{i,j}$ - value of gas pressure at *l+0.5* -th time layer; $\hat{P}_{i,j}$ - value of gas pressure at the *l* -th time layer.

Let us introduce the following designation:

$$T_{i-0.5,j} = \frac{k_{i-0.5,j} h_{i-0.5,j}}{\mu}. \quad T_{i+0.5,j} = \frac{k_{i+0.5,j} h_{i+0.5,j}}{\mu}.$$

$$T_{i,j-0.5} = \frac{k_{i,j-0.5} h_{i,j-0.5}}{\mu}. \quad T_{i,j+0.5} = \frac{k_{i,j+0.5} h_{i,j+0.5}}{\mu}.$$

Obviously, the obtained difference equations (9) and (10) with respect to the pressure function P are nonlinear. Therefore, an iterative method based on the method of quasi-linearization of nonlinear terms [3-6,8] is used for the solution.

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According to this method, the nonlinear terms of the difference equation can be represented in the following form:

$$\psi(P) \cong \psi\left(\mathring{P}\right) + \left(P - \mathring{P}\right)\frac{\partial \psi\left(\mathring{P}\right)}{\partial P}. \quad (11)$$

Here $\mathring{P}$ is the approximate value of the pressure function P, which is refined during the iteration process $\mathring{P} = P_{i,j}^{(s)}$, while $P_{i,j}^{(0)} = \hat{P}_{i,j}$. $\hat{P}_{i,j}$ - initial value of the pressure function.

If formula (11) is written for the square of the pressure function $P^2 \approx 2\mathring{P}P - \mathring{P}^2$, then instead of the finite-difference equation (9) we obtain the following equation [9,10]:

$$2T_{i-0.5j}\mathring{P}_{i-1j}\bar{P}_{i-1j} - \left[\left(T_{i-0.5j} + T_{i-0.5j}\right)\mathring{P}_{ij} + \frac{\Delta x^2}{\tau/2}\right]\bar{P}_{ij} + 2T_{i+0.5j}\mathring{P}_{i+1j}\bar{P}_{i+1j} =$$

$$= \left[T_{i-0.5j}\mathring{P}^2_{i-1j} - \left(T_{i-0.5j} + T_{i+0.5j}\right)\mathring{P}^2_{ij} + T_{i+0.5j}\mathring{P}^2_{i+1j}\right] -$$

$$-\frac{(\Delta x)^2}{(\Delta y)^2}\left[T_{ij-0.5}\hat{P}^2_{ij-1} - \left(T_{ij-0.5} + T_{ij+0.5}\right)\hat{P}^2_{ij} + T_{ij+0.5}\hat{P}^2_{ij+1}\right] - \frac{(\Delta x)^2}{\tau/2}\hat{P}_{ij} -$$

$$-(\Delta x)^2 \delta_{i,j} q_{i,j}.$$

Here $\bar{P}_{i-1,j}, \bar{P}_{i,j}, \bar{P}_{i+1,j}$ are the values of the pressure function in the $l+0.5$ nd time layer; $\mathring{P}_{i-1,j}, \mathring{P}_{i,j}, \mathring{P}_{i+1,j}$ - approximate values of the pressure function; $\hat{P}_{i,j-1}, \hat{P}_{i,j}, \hat{P}_{i,j+1}$ - values of the pressure function in the *l*-th time layer.

Approximating the boundary conditions (7) and from the last finite difference equation, we obtain the following three-point difference equations for the $l+0.5$ nd time layer:

$$\begin{cases} 2\Delta x L\bar{P}_{0,j} + 4\bar{P}_{1,j} - \bar{P}_{2,j} = 2\Delta x L P_A, \\ a_i\bar{P}_{i-1,j} - b_i\bar{P}_{i,j} + c_i\bar{P}_{i+1,j} = -d_i, \quad i = 1,2,...,N-1, \\ 2\Delta x L\bar{P}_{N,j} + 4\bar{P}_{N-1,j} - \bar{P}_{N-2,j} = -2\Delta x L P_A. \end{cases} \quad (12)$$

Where $a_i = 2T_{i-0.5,j}\mathring{P}_{i-1,j}, \quad c_i = 2T_{i+0.5,j}\mathring{P}_{i+1,j}, \quad b_i = a_i + c_i - \frac{(\Delta x)^2}{\tau/2},$

$$d_i = \frac{(\Delta x)^2}{\tau/2}\hat{P}_{i,\mathrm{j}} - \left[T_{i-0.5,j}\tilde{P}_{i-1,j} - \left(T_{i-0.5,j} + T_{i+0.5,j}\right)\tilde{P}_{i,j} + T_{i+0.5,j}\tilde{P}_{i+1,j}\right] -$$

$$-\left[T_{i,j-0.5}\hat{P}^2_{i,j-1} - \left(T_{i,j-0.5} + T_{i,j+0.5}\right)\hat{P}^2_{i,j} + T_{i,j+0.5}\hat{P}^2_{i,j+1}\right] - \delta_{i,j}q_{i,j}.$$

Similarly, using the scheme of alternating directions in the variable y for the partial differential equation (5) and approximating the boundary conditions (7), we obtain the following three-point difference equation for the *l+1* -th time layer:

$$\begin{cases} 2\Delta yLP_{i,0} + 4P_{i,1} - P_{i,2} = 2\Delta yLP_A, \\ a_jP_{i,j-1} - b_jP_{i,j} + c_jP_{i,j+1} = -d_j, \ \ j = 1,2,...,M-1, \\ 2\Delta yLP_{i,M} + 4P_{i,M-1} - P_{i,M-2} = -2\Delta yLP_A. \end{cases} \qquad (13)$$

where $a_j = 2T_{i,j-0.5}\tilde{P}_{i,j-1}, \ b_j = a_j + c_j - \frac{\Delta y^2}{\tau/2}, \ c_j = 2T_{i,j+0.5}\tilde{P}_{i,j+1},$

$$d_j = \frac{\Delta y^2}{\tau/2}\bar{P}_{i,\mathrm{j}} - \left[T_{i,j-0.5}\tilde{P}_{i,j-1} - \left(T_{i,j-0.5} + T_{i,j+0.5}\right)\tilde{P}_{i,j} + T_{i,j+0.5}\tilde{P}_{i,j+1}\right] -$$

$$-\left[T_{i-0.5,j}\bar{P}^2_{i-1,j} - \left(T_{i-0.5,j} + T_{i+0.5,j}\right)\bar{P}^2_{i,j} + T_{i+0.5,j}\bar{P}^2_{i+1,j}\right] - \delta_{i,j}q_{i,j}.$$

The obtained finite-difference equations (12) are solved on $l+0.5$ nd time layer, and equation (13) - on the $l+1$ nd time layer by the sweep method.

As can be seen, the accuracy of approximation of the equation and boundary conditions is of the order $O\left(\tau, (\Delta x)^2, (\Delta y)^2\right)$.

The iterative process continues until the condition

$$\max_{i,j} |\, P^{(s)}_{i,j} - P^{(s-1)}_{i,j} \,| \le \varepsilon \,, \qquad (14)$$

where ε - iteration accuracy, a predetermined small value;

s - number of iteration.

Based on the numerical model, a solution algorithm and a calculation program were developed, with the help of which a number of computational experiments were carried out on a PC to study the processes of gas filtration in a porous medium.

Numerical algorithm. The two-dimensional boundary value problem of gas filtration in a piecewise inhomogeneous porous medium described above can be easily converted into a finite difference problem using a longitudinal-transverse scheme and solved by sweep methods. In this case, the numerical implementation of the algorithm for solving the problem for a fixed time layer is carried out at the following stages.

The first stage of the algorithm for calculating the main indicators of the development of a gas field is carried out by calculating the values of the pressure function on the $l+0.5$ nd time layer in the direction of the variable *x* with a fixed variable *y*. In this case, the method of sweeping in the direction of the variable *x* is used. The calculations are carried out as follows:

1. Entering the initial data.
2. Determining the initial values of the fitting coefficients α_0 and β_0 from the boundary conditions of the left side of the discrete filtration area, i.e. from the first equation of the three-point system (12);
3. Calculation of the coefficients a_i, b_i, c_i, d_i $\left(i=1,..,N-1\right)$ of the three-point difference equation (12);
4. Calculation of the value of the sweep coefficients α_i and β_i $\left(i=1,..,N-1\right)$;
5. Determining the final values of the pressure function P_N , from the boundary conditions of the right side of the discrete filtration area, i.e. from the third equation of the three-point system (12);
6. Backward sweep calculates the values of the pressure function P_i $\left(i=N_j-1, N_j-2,..,1\right)$.
7. Checking the condition of the iterative process according to the formula (14).
8. If the condition of the iterative process is met, then the transition to the *l + 1* -th time layer is carried out, i.e. the transition is carried out to 2. In this case, the initial condition and iteration for the *l + 1* -th time layer will be only the found valued pressure $P_{i,j}$.

If the iteration conditions (14) are not satisfied, the transition is carried out to 2. In this case, the new iteration values are determined by the pressure function $P_{i,j}^{(s+1)}=P_{i,j}^{(s)}$.

9. A similar procedure (2-6) is repeated in the second stage for the *l+1* -th time layer.
10. Checking the condition of the iterative process according to the formula (14).
11. If the condition of the iterative process is met, then the transition to the $l+1$ -th time layer is carried out, i.e. the transition is carried out to 2. In this case, the initial condition and iteration for the $l+1$ -th time layer will be only the found valued pressure $P_{i,j}$.

If the iteration conditions (14) are not satisfied, the transition is carried out to 2. In this case, the new iteration values for the function are determined by $P_{i,j}^{(s+1)}=P_{i,j}^{(s)}$.

12. Output of the final results in graphical form.
13. The end of the calculation.

Computational experiment. To solve the problem of gas filtration in a piecewise inhomogeneous porous medium, software has been developed for carrying out computational experiments that allow the visualization of the numerical results of the calculation, displaying in the form of a 3D graph the main indicators of the development of gas fields.

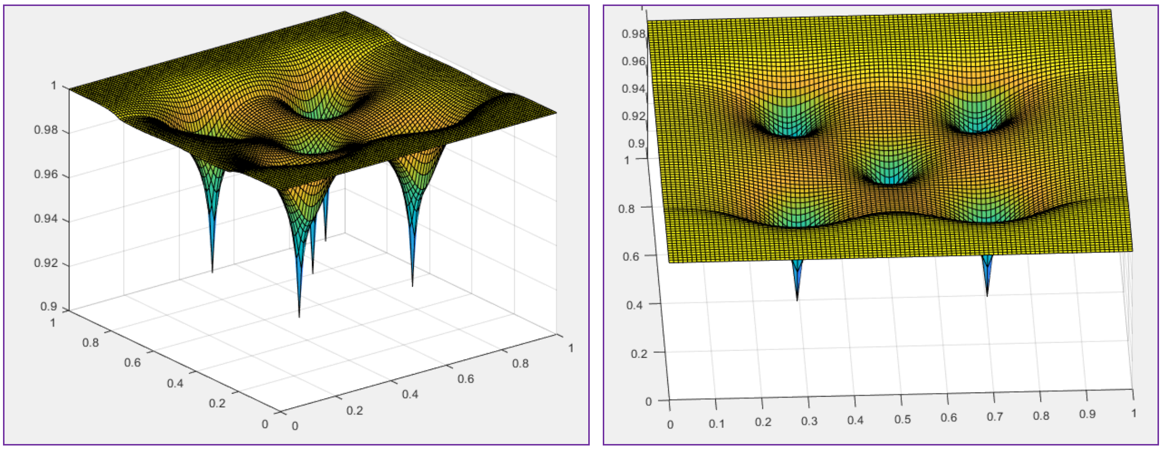
For the purpose of computational analysis, the values of the parameters to be included in the program for the filtration layer gas were obtained as follows (Table 1).

Table 1.

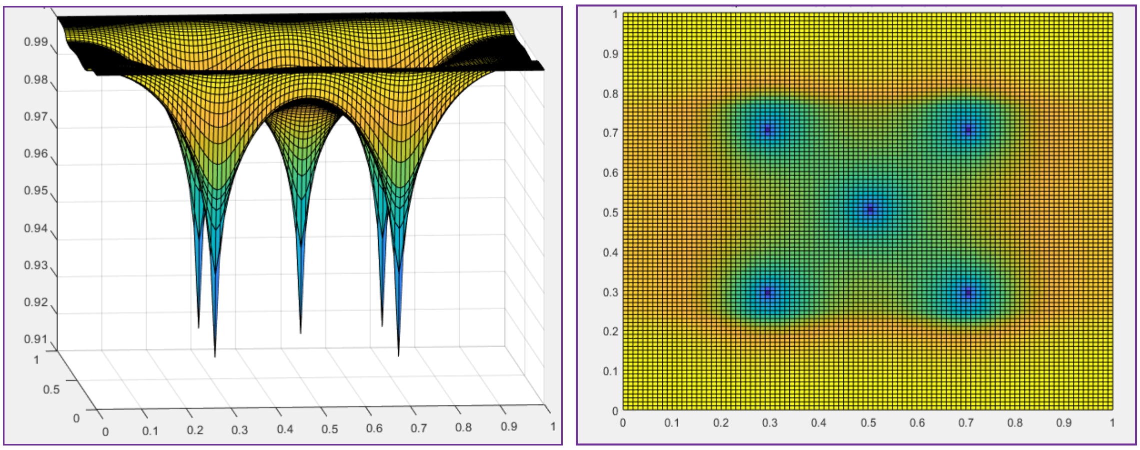
Initial data on layer and gas characteristics

Name of initial data	Notation of input parameters	Numerical value and dimension of parameters
Initial formation pressure	P	300 kg/sm^2
Layer thickness	h	10 m^2
Gas viscosity	μ	0.01, 0.03 sP
Layer permeability	K	0.05, 0.3 Darcy
Gas saturation coefficient	a	0.2
Well flow rate	$q_Г$	200000 m^3/day
Layer porosity	m	0.1
Characteristic length of the region	L	8 km
Iteration Accuracy	ε	0.0001
Ultimate development time	t	1444 day

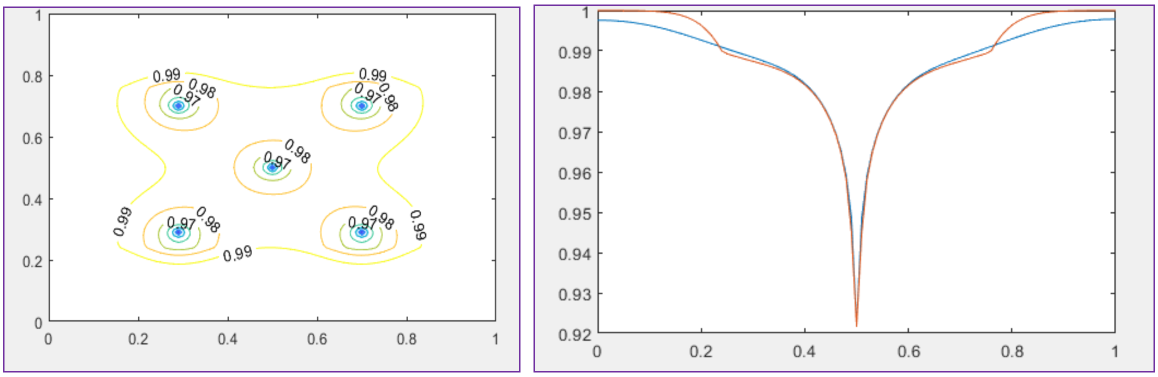
The considered non-stationary gas filtration in a piecewise heterogeneous layer is developed by a system of wells with a given constant flow rates. The results of computational experiments are shown according to the distribution of pressure in the layer and the drop in the wells in pic. 1-4 in visual form.



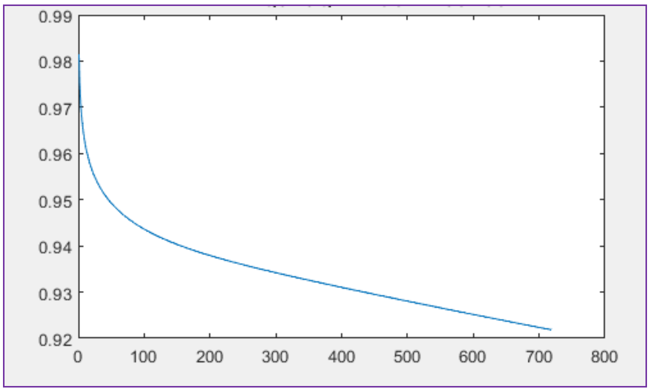
Pic. 1. Pressure distribution in a gas reservoir 3D graph (view from the top angle and from the top side)



Pic. 2. Pressure distribution in a gas deposit 3D graph (side and top view)



Pic. 3. Distribution of pressure in the gas reservoir in the contour plot and in the section (x=0.5 and y=0.5)



Pic. 4. Pressure drop in the central well

The computational experiment was carried out for different values of the layer permeability coefficient at the development time of 1444 days. The results obtained show that the layer permeability coefficient significantly affects both the dynamics of the pressure distribution in the layer.

In the computational experiment, the value of the layer conductivity coefficient was introduced as follows:

$$k(x) = \begin{cases} 0.05 & 0 \le x \le 0.3, \\ 0.3 & 0.3 < x < 0.7, \\ 0.05 & 0.7 \le x \le 1. \end{cases}$$

Due to the location of the well in the center of the symmetrical view and the values of the area of the formation permeability coefficient along the abscissa, it can be seen from the result that there is a break in the graphs at points 0.3 and 0.7 on the *x* axis. This shows that at these points the coefficient *k(x)* of the equation changes its value by a large difference.

The iterative process continues at the iteration accuracy value ε=0.0001 approaches at s=4-5 iteration number.

Conclusion. The developed numerical methods for calculating the main indicators of gas field development can be used in the analysis and design of field development.

The results of computational experiments clearly confirm the graph of areal pressure changes and pressure distribution in sections, as well as wells.

References

1. Abutaliev F.B. Efficient Approximately Analytic Methods for Solving Problems in the Theory of Filtration. Ed. “Fan”, Tashkent-1978.

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10.5281/zenodo.7335157
2. Abutaliev F.B., Khadzhibaev N.N., Izmailov I.I., Umarov U. Application of numerical methods and computers in hydrogeology. Publishing house “Fan”, Tashkent -1976.

3. Agafonov S.A., German A.D., Muratova T.V. Differential equations. - MSTU im. N.E. Bauman, 2004. -348 p. - (Mathematics at the Technical University)

4. Aziz H., Sattari E. Mathematical modeling of reservoir systems. Maskva-Izhevsk. 2004.

5. Belman R., Calaba R. Quasi-linearization and nonlinear boundary value problems. Mir, M., 1968.

6. Gusein-Zade M. A., Kolosovskaya A. K. Elastic regime in single-layer and multi-layer systems. M., "Nedra", 1972, 454 p.

7. 7. Zakirov S.N., Lapuk B.B. Design and development of gas fields. Ed. Nedra, M. 1974.

8. Mukhidinov N. Methods for calculating indicators for the development of multilayer oil and gas fields. // Tashkent, ed. "Fan", 1978, 117p.

9. Nematov A., Nazirova E.Sh. Numerical modeling of the process of gas filtration in porous media // International Academic Bulletin. - 2016. - No. 1(13). - P. 52-56.

10. Ravshanov N., Kurbonov N.M. Numerical modeling of the process of gas filtration in a porous medium // Information technologies of modeling and control. - Voronezh, 2016. - No. 1 (97). - P. 34-45.

11. Samarsky A.A. Theory of difference schemes. - M.: Nauka, 1977. – 656 P.

12. A Nematov, E Sh Nazirova and R T Sadikov On numerical method for modeling oil filtration problems in piecewise-inhomogeneous porous medium // CIEES 2020 IOP Conf. Series: Materials Science and Engineering 1032 (2020) 012018. Bulgaria Volume 1032 (012018), – P. 1-7. (Scopus; IF=0.7)