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METHODS OF CONSTRUCTING 3D SHAPES OF HYPERCOMPLEX FRACTALS

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Abstract. This article is about building fractals and designing 3D shapes. The proposed methods allow users to easily edit fractals in three ways: changing the shape and parameters of the fractals, and changing the angles to create a different shape. Examples of the Menger sponge, Serpin tetrahedron and Mandelbulb fractals have their shape controlled by the Kaleidoscopic effect, and 3D fractal shapes are constructed as variable geometric objects. The "Fraktal me'mor" software tool presented in this article made it possible to develop many attractive forms of kaleidoscopic hypercomplex fractals by changing the flexible parameters of the developed fractals.

Keywords: Fractal, fractal geometry, design, construction, architectural form, geometry, model, kaleidoscopic effect, IFS, hypercomplex.

Introduction

Although the term "fractal" was introduced to science in the 20th century, similarities in geometry were known to people much earlier [1]. Fractals are not recognized as a major branch of mathematics, but fractals are constantly used in architecture, art, medicine, and other fields. Artists can use their infinite recursion to create beautiful, unnatural, artistic forms that are the basis of Fractal art. In doing so, he uses the theory of fractals to describe the art developed in computer graphics. Currently, hyper-complex fractals are appearing in feature animation, landscape design, architecture, and other visual media. It should be noted that fractal art is mainly 2D, because it is difficult to control the algorithms that generate fractals in 3D.

In architecture and other fields of research, there are ideas about the methods of constructing fractals and its measurement [2-7].

There are many approaches to constructing fractals. For example, the construction of classical Cantor sets of fractal equations consisting of spirals based on the theory of R-functions (RFM) using recursion procedures. According to the created equations, different prefractals are generated depending on the number of iterations. All results are shown as a raster diagram [2].

The fractal dimension was introduced as a coefficient describing geometrically complex shapes, for which the importance of details rather than the full drawn image reflected the capabilities of the method. These studies further develop the method of calculating cells of fractal analysis and allow not only to measure the complexity of the properties of the measured objects, but also to check the consistency of the properties of the elements of these objects by comparing the relationship between their properties [3].

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A detailed depiction of the different mathematical methods for deciding the dimensions of complex geometric objects with a fractal structure and the investigation of errors in determining the fractional measure of complex geometric objects are displayed. The article presents the concept of fractal estimation, properties, topological estimation, estimations of designs and scenes in nature, differences between Hausdorf-Bezikovich measurement and Mandelbrot-Richardson measurement, fractal measurements. Dimensions of complex geometric objects with several fractal structures have too been identified [4].

Basic concepts of the theory of fractals, areas of application and their types have been presented. The basic methods of constructing fractals are taken into account: L-system method, system of iterating functions, set theory method, and the R-function method. Equations of complex structures of fractal geometry have been developed based on the R-functions method [5].

The phenomena of geophysical origin of fractal geometric shapes and potential useful tools for describing complex shapes are illustrated. Despite, the fractals are widely used in geographical areas; the opinions which cause inconsistent results from different fractal computational algorithms have been expressed. Fractal dimension was firstly introduced as the coefficient which describes geometrically complex shapes, the details are considered more important than a completely drawen picture [6].

This article focuses on the mathematical modeling of triangular fractal patterns and carpet design in textiles, demonstrating the classic and new arithmetic, combinatorial features of the generalized arithmetic triangles of Pascal's triangle. Binomial, three-term, and other combinatorial numbers constructed based on recurrence relations have been investigated. The software "Pascal's Triangle" was developed in the C# programming language. A convenient user interface of the program has been developed. Based on the software implementation, arithmetic triangles are constructed from residues in the form of Pascal's triangle and combinatorial numbers. The developed software can be used in industry, when drawing patterns, for their subsequent stamping on carpets, fabrics, ceramic tiles, etc [7].

The article explores methods of constructing hypercomplex fractals and presents a method for developing attractive shapes. Extending the definition of the 3D Mandelbulb, Serpin Tetrahedron and Menger Sponge algorithms with the next self-defined kaleidoscopic effect, many different fractal shapes are developed simultaneously allowing artistic control. Implementation of the algorithm using the "Fraktal me'mor" software tool allows the user to develop unique 3D shapes. There are several 3D models to see the application of the algorithm to fractals that use very complex numbers in their definitions, notably the 2D Mandelbrot, Serpin Triangle, Serpin Napkin, and 3D Mandelbulb, Serpin Tetrahedron, and Dedicated to Menger sponge.

Main part

The Mandelbrot set is constructed by repeating a simple recursive rule with a complex number, whose pattern appears in the complex plane. It was first discovered

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in 1978 by Robert W. Brooks, who drew it for research on Klein groups [9]. The mathematical definition of the fractal set was made by Benoit Mandelbrot in 1980. The Mandelbrot set is defined by equation [10].

$$z_{n+1} = z_n^2 + c \quad (1)$$

Here, z and c are complex numbers, and if $|z|<2$ for every n iteration, the point is said to belong to the set. The Mandelbrot set divides the set of complex numbers into two subsets, and when the fractal visualization is enriched with color, its properties can be better recognized later. For example, the colors of the points are determined by the number of iterations needed to reach $r_{max}=2$, where $r=|z|$. In Figure 1, you need to repeat "0" for red dots, "1" for pink dots, "2" for purple dots, "3" for blue dots or more.

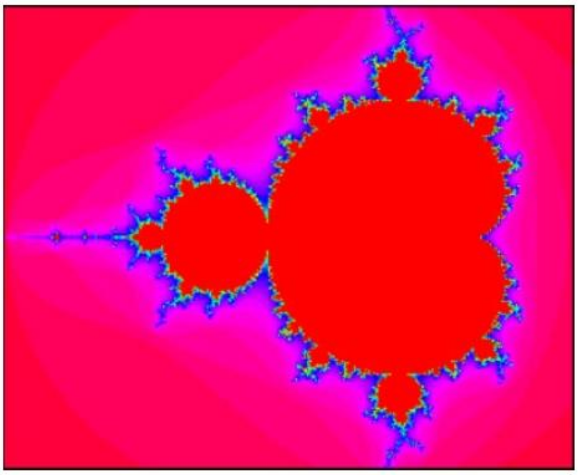

Fig 1. Mandelbrot set

There are different ways to convert the Mandelbrot set from 2D to 3D. One such method is to rotate the set around its axis. The most famous “Mandelbrot 3D” is the Mandelbulb, discovered by Daniel White [11], in which the 2D to 3D transformation is done by converting Cartesian coordinates to spherical coordinates and then transforming them back. This algorithm uses formula (1) like the Mandelbrot set, with the difference that z and c are hypercomplex (triplex) numbers representing 3D Cartesian coordinates. It is defined as an exponential term in its definition:

$$\langle x, y, z\rangle^n = r^n \langle \cos n\phi \sin n\theta, \sin n\phi \sin n\theta, \cos n\theta \rangle \quad (2)$$

Here $r = \sqrt{x^2 + y^2 + z^2}$, n is the order, $\theta = \operatorname{atan2}\left(\frac{z}{\sqrt{x^2 + y^2}}\right)$ and $\theta = \operatorname{atan2}\left(\frac{y}{x}\right)$ are defined as angles. A Mandelbulb is a collection of 3D points defining an object of finite volume, whose infinite recursion results in a complex surface area. We often call the Mandelbulb, Serpin's pyramid, and Menger's sponge “Hypercomplex Fractals”. This is because Mandelbrot sets use complex numbers, while Mandelbulb, Serpin's pyramid, and Menger sponges use supercomplex numbers, which are similar to complex numbers but expanded in 3D [12]. Fractal art. Fractals were used in

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architecture and art, such as mosques and churches, long before the term "fractal" was coined. However, fractal art has mostly used 2D, with a few exceptions such as fractal drawings [8] and relief carving [13]. The main reason why 2D fractals are used for artistic purposes is that the algorithms that generate fractals in 3D format are difficult to control. In recent years, hypercomplex fractals have become an integral part of visual effects. They have been used in many feature films. Big Hero 6 [14] used a variation of the Mandelbulb algorithm with parameters that allow for easy creation of various 3D shapes. Similarly, the “Fraktal me’mor” software tool has editable parameters.

In 2006, an artwork called “Fractal Effervescence” was written by David April to develop fractal patterns and visual images based on mathematical formulas. The artwork was created by combining three image files created using the Apophysis software developed by Mark Townsend. The three files had different types of transformations which created this image [15].

Johan Andersson created unusual fractal jewelry and accessories using 3D fractals in 2009, shortly after Daniel White discovered the Mandelbulb [16]. One of his works was creating fractal art by 3D printing unusual chess pieces. His work is based on transforming mathematical algorithms into hypercomplex fractals to develop 3D fractal shapes [17].

“Fraktal me’mor” is a software tool developed for the construction of 2D and 3D fractal shapes within the framework of research. It has a node-based workflow that is ideal for visual effects, as it allows users to create dynamic simulations, as it is designed for visual effects, and its custom nodes allow us to quickly visualize mathematical algorithms.

Methods

As mentioned above, one of the main goals of this research is to create complex fractal shapes as the main component of visual elements and at the same time effectively visualize the generated fractal shapes. Mandelbulb's surface area is infinite, so it approximates a highly complex fractal shape by representing the object in a high-resolution three-dimensional image. Despite some flexibility in the Mandelbulb definition, transformations can be achieved by n-order corrections (see equation 1), but there are still limitations in the resulting form. The Mandelbulb formula (Equation 1) was modified as part of the experiment to increase the variation. The first modification used to get an "inverted" Mandelbulb. The resulting shape is significantly different from the original Mandelbulb, as shown in Figures 2(a) and 2(b).

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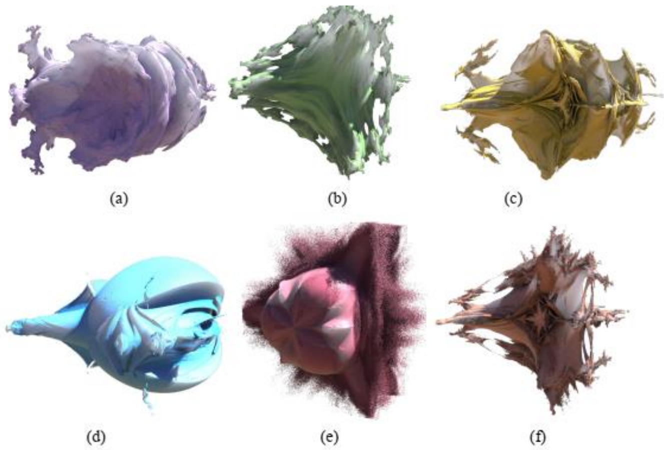

Fig.2. Supercomplex fractals of order 2 rendered by “Fraktal me’mor” software: (a) original Mandelbulb, (b) inverted Mandelbulb, (c, d) deformed inverted Mandelbulb, (e) modified Mandlebulb before rotation, (f) adjusted after rotation Mandelbulb with transformation.

A self-defined kaleidoscope effect has been added to allow more customization of the resulting shape [9]. As mentioned above, hypercomplex fractals appear infinitely complex on the surface. However, it consists of repeating patterns, although they are not easily recognizable by the human eye. We combined this with a kaleidoscope to see the effect it creates. Parts of the fractal are imported using the Fraktal me’mor's Build button and mirrored from multiple sides using user-defined copy buttons. Because of this method, a kaleidoscopic Mandelbulb can only have even sides, otherwise one part of the Mandelbulb will not have a mirrored version and therefore will not connect properly to the rest of the mesh. Kaleidoscopic Mandelbulbs can also be edited to the user's preference by simply rotating the Mandelbulbs to select which part of the Mandelbulb is displayed. This allows us to develop many different kaleidoscopic Mandelbulbs of different shapes. This produced some dramatic changes between the fractals, even though they were all created from the same equation, as shown in Fig.3.

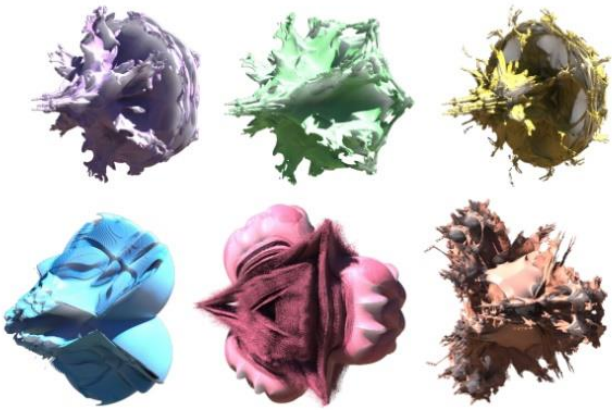

Fig.3. Kaleidoscopic effect applied to the Mandelbulbs in Figure 2

The Mandelbulb image in Figure 2 is shown in Figure 3 after applying the kaleidoscope effect. The mandelbulbs are bent in such a way that the kaleidoscope

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effect creates a pleasing sight to the eye. With this modification, you can create many hypercomposite fractal shapes by slightly changing the parameters [11].

Results

First of all, hypercomplex two- and three-dimensional fractal shapes were developed based on algorithms proposed and improved through the C++ programming language. After obtaining positive results, C++ + Qt 6 + OpenGL technology was used based on the existing proposed and improved algorithms, and the architecture of the software tool was developed using the C++ programming language (Fig. 4). “Fraktal me’mor” software was used to develop fractals because it allows for quick visualization. This software tool was developed based on the Mandelbulb fractal formula and Inigo Quilez algorithm mentioned by Daniel White and Paul Nylander [5]. The advantage of this program is that the program is not limited to the Mandelbulb fractal formula, but also allows to visualize fractal forms such as Menger sponge, Serpin tetrahedron (Fig. 5).

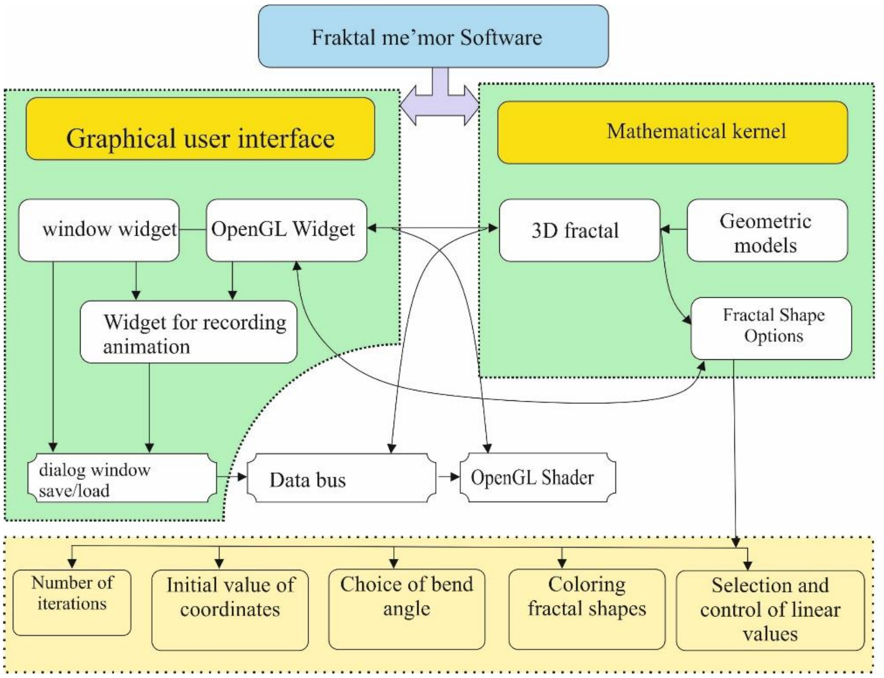

Fig. 4. Software architecture

The vex code converts the Cartesian coordinates of a point in the volume to the spherical coordinates.

(a)

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(b)

Fig. 5. The initial state of the Menger sponge (a), the Serpin tetrahedron (b) and the shapes created after applying the kaleidoscopic effect

Conclusion

Fractal art is a field that combines mathematics and digital art, as it allows the creation of infinitely complex shapes with very simple formulas. In this work, it was shown how easy modifications of Menger sponge, Serpin tetrahedron fractals can be converted into a volumetric object and then lead to many fractal shapes that can be implemented as a 3D geometric model. This was achieved by changing the formula in the “Fraktal me’mor” software tool, applying a kaleidoscopic effect. The results are aesthetically appealing and a first step into fractal art, with scope for further development of the idea and application to architecture. Another important aspect to consider is the artistic control of fractal shapes. For example, it would be interesting to study how an architect can make changes in fractal space and when the shape of a fractal is defined. In short, it can be expected that it can be done by manually drawing over the fractal shape, but at the same time, investigating semi-automatic ways of doing it will also be a direction for future research. To make the final shapes more aesthetically pleasing, Menger's sponge, coloring the Serpinski tetrahedron in colors allows for a brighter appearance.

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