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NUMERICAL METHOD FOR SOLVING THE PROBLEM OF FILTRATION OF UNSTABLE GROUNDWATER AND AN EFFECTIVE COMPUTING ALGORITHM

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Abstract. This article deals with the numerical method and efficient calculation algorithm of solving the unstable groundwater filtration problem, numerical methods of solving the boundary value problem set to the parabolic differential equation, numerical modeling using the finite-difference method, and the development of parallel computing algorithms.

Keywords: differential equation, parallel computation, parallel algorithm, pressurized water, efficiency, finite-difference method, mathematical model, discrete model, experimentation.

Introduction. The main method of growing agricultural crops in Central Asia is artificial irrigation. Much attention is paid to the widespread use of groundwater, which is an internal reserve, since the irrigation process is limited to water resources for the development of Agriculture.

It is known that the development of irrigated areas in agriculture is limited by water resources, so much attention is paid to the widespread use of groundwater, which is an internal reserve. The construction of hydrodynamic and reclamation facilities, the use of groundwater in water supply and irrigation, contribute to a change in the balance and regime of groundwater. Therefore, all explotational reserves of groundwater should be based on a set of dynamic assessment studies. A comprehensive study of the filtration process of groundwater in a non-homogeneous layer with unstable pressure is of great interest in a comprehensive study and assessment of the use of groundwater reserves.

One of the only ways to carry out comprehensive research on the filtration process of groundwater is the mathematical modeling of the mode of operation of aquifers using various mathematical models of Filtration Theory.

The choice of optimal mathematical models of Filtration Theory cannot be imagined without a complete analysis of quantitative estimates, various natural and artificial factors that affect the process under study. It is advisable to carry out such an analysis using a digital (numerical) experiment using a personal computer.

This process consists of the following steps:

1. Problem formulation; mathematical model;
2. Calculation algorithm;
3. Programming;
4. Computing experience.

The scientific research work carried out in this area pays great attention to the development of mathematical models and effective numerical algorithms, as well as software. Due to the fact that the research of a complex filtration process in the

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effective use of groundwater takes a lot of time, it is carried out only through the use of modern computer technology.

It is known that modern computer systems are multi - processor computers with different configurations with different computing nodes-with different frequencies, different volumes of memory and the same architecture. Such computers, firstly, will further increase the level of parallel computing, and secondly, non-heterogeneity will increase the possibility of a parallel computing system.

In order to numerical modeling of the issue posed when solving practical issues of a particular area, as well as to increase the efficiency of computing on computers, it will be necessary to create new well-yielding parallel computing algorithms.

Even today, there is a great deal of attention to the creation of many mathematical models, calculation algorithms and software for the issue of filtration in porous environments for the research of the filtration process of groundwater.

Mathematical model. Let us consider the movement of unstable underground pressurized waters in the horizon of a non-same-sex filtration area with optional borders and Wells. Γ has a limit G in the filtration field, the mathematical model of pressurized water motion is described by the following parabolic-type differential equation:

$$\mu^* \frac{\partial h}{\partial t} = \frac{\partial}{\partial x}(T\frac{\partial h}{\partial x}) + \frac{\partial}{\partial y}(T\frac{\partial h}{\partial y}) - q; \ \ (x,y) \in G \tag{1}$$

$$h(x,y,t_0) = F_1(x,y);(x,y) \in G, \tag{2}$$

$$\left.\begin{aligned} & h(x,y,t) = F_2(x,y,t);(x,y) \in \Gamma_i; t \geq t_0; \\ & \frac{\partial h}{\partial n}(x,y,t) = F_3(x,y,t);(x,y) \in \Gamma_j; t \geq t_0; \end{aligned}\right\} \tag{3}$$

$$\sum_{i=1}^{m_1} \Gamma_i + \sum_{j=1}^{m_3} \Gamma_j = \Gamma,$$

$$\int_{\Gamma_k} T\frac{\partial h}{\partial n} ds = Q_k(t); t \geq t_0; k = 1,\overline{m}_3 \tag{4}$$

Here

T(x,y)=Km;

h (x,y,t) - pressurized water rise;

μ^* , *K(x,* y) - coefficient of water supply and permeability coefficient of the layer;

m - power of the water source horizon;

q(x, y, t) - water well;

$\Gamma_к$ - contour;

Q_k(t) - the well debit;

ds -$\Gamma_к$ element ;

m_3 - number of wells;

$F_1(x,$ y); F_2(x, *y,t)*; $F_3(x, y, t)$ - given functions.

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To obtain a numerical solution to the boundary value problem, which is formulated in terms (1) and (2), (3), let's move on to dimensionless variables.

$$h^* = \frac{h}{h_0};\ T^* = \frac{T}{T_0};\ x^* = \frac{x}{L};\ y^* = \frac{y}{L};\ \tau = \frac{T_0 t}{\mu L^2};\ q^* = \frac{L^2 q}{m_0 T_0},$$

here h_0, T_0 -some characteristic values of functions $h\ (x,\ y,\ t)$ and $T\ (x, y)$;
L -maximum length of the layer.
We will have a dimensionless differential equation as follows:

$$\frac{\partial h}{\partial \tau} = \frac{\partial}{\partial x}(T\frac{\partial h}{\partial x}) + \frac{\partial}{\partial y}(T\frac{\partial h}{\partial y}) + q \qquad (5)$$

(5) asterisks in dimensionless variables were removed for convenience.

Numerical modeling. To solve the problem posed in numerical ways, we build the following mesh:

$$\Omega_{xy\tau_k} = \left\{ \left(x_i = ih_{xy},\ y_j = jh_{xy},\ \tau_k = k\ \Delta\tau \right);\ i = \overline{1,N};\ j = \overline{1,M},\ k = 1,2,3,...;\ \Delta\tau = \frac{1}{N_t} \right\}$$

To obtain a finite subtraction problem, the algorithmic idea of an undisclosed scheme of variable directions is used, which makes it possible to apply the driving method along each straight coordinate line. the transition from layer k to layer k+1 is carried out in two stages with a step of 0.5 t. As a result, a sequential solution of a system of equations with two numerical of differences is formed.

The finite-subtraction equation in the time layer k+0.5 is as follows:

$$\frac{\Delta h^2}{\Delta\tau/2}\left(h_{i,j} - \hat{h}_{i,j}\right) = \left(T_{i-0,5,j}h_{i-1,j} - \left(T_{i-0,5,j} + T_{i+0,5,j}\right)h_{i,j} + T_{i+0,5,j}h_{i+1,j}\right) +$$
$$+\left(T_{i,j-0.5}\hat{h}_{i,j-1} - \left(T_{i,j-0.5} + T_{i,j+0.5}\right)\hat{h}_{i,j} + T_{i,j-0.5}\hat{h}_{i,j+1}\right) - q_{iq}$$

Here:
$\hat{h}$ - pressure, k in the time layer;
h - pressure, k+0.5 in the time layer.
In this case, the coefficients of the finite subtraction equation will have the following appearance:

$$a_i = T_{1i-0,5},\ \ c_i = T_{i+0,5,j},$$
$$b_i = a_i + c_i + \frac{h_{xy}^2}{\Delta\tau/2}h_{i,j},$$
$$d_i = \frac{h_{xy}^2}{\Delta\tau/2}\hat{h}_{i,j}.$$

Similarly for the time layer k+1, an equation with a finite subtraction is constructed. By opracimating the boundary conditions, we get the following system of finite subtraction equations for the time layer k+0.5.

$$3h_{0,j} - 4h_{1,j} + h_{2,j} = 0;\ , \qquad (6)$$

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$$a_i h_{i-1,j} - b_i h_{i,j} + c_i h_{i+1,j} = -d_i, \quad i = 1, 2, .., N-1, \tag{7}$$

$$3h_{N,j} + 4h_{N-1,j} - h_{N-2,j} = 0. \tag{8}$$

System of equations with the resulting finite subtraction (6), (7), (8) are solved using the driving method. for the time layer k+1, too, the calculation scheme is carried out in the same way.

The parallel calculation algorithm to be described is based on the idea of a scheme of directions of variables (cross-sectional scheme) and a meeting-driving method. As you know, the combination of the left and right driving method gives a meeting-driving method, which allows to us parallelize two streams.

In this case, the finite-subtraction system is divided between two streams - the first $1 \le i \le \bar{n}$ and the second $\bar{n} \le i \le n$ (where $\bar{n} = (n+1)/2$, n is odd). Here $\bar{n}$ is the number of the equation in which the two branches of the forward move “meet” – “above” and “below”.

A block-scheme drawing of a parallel calculation algorithm based on a cross-sectional scheme is shown in Figure 1.

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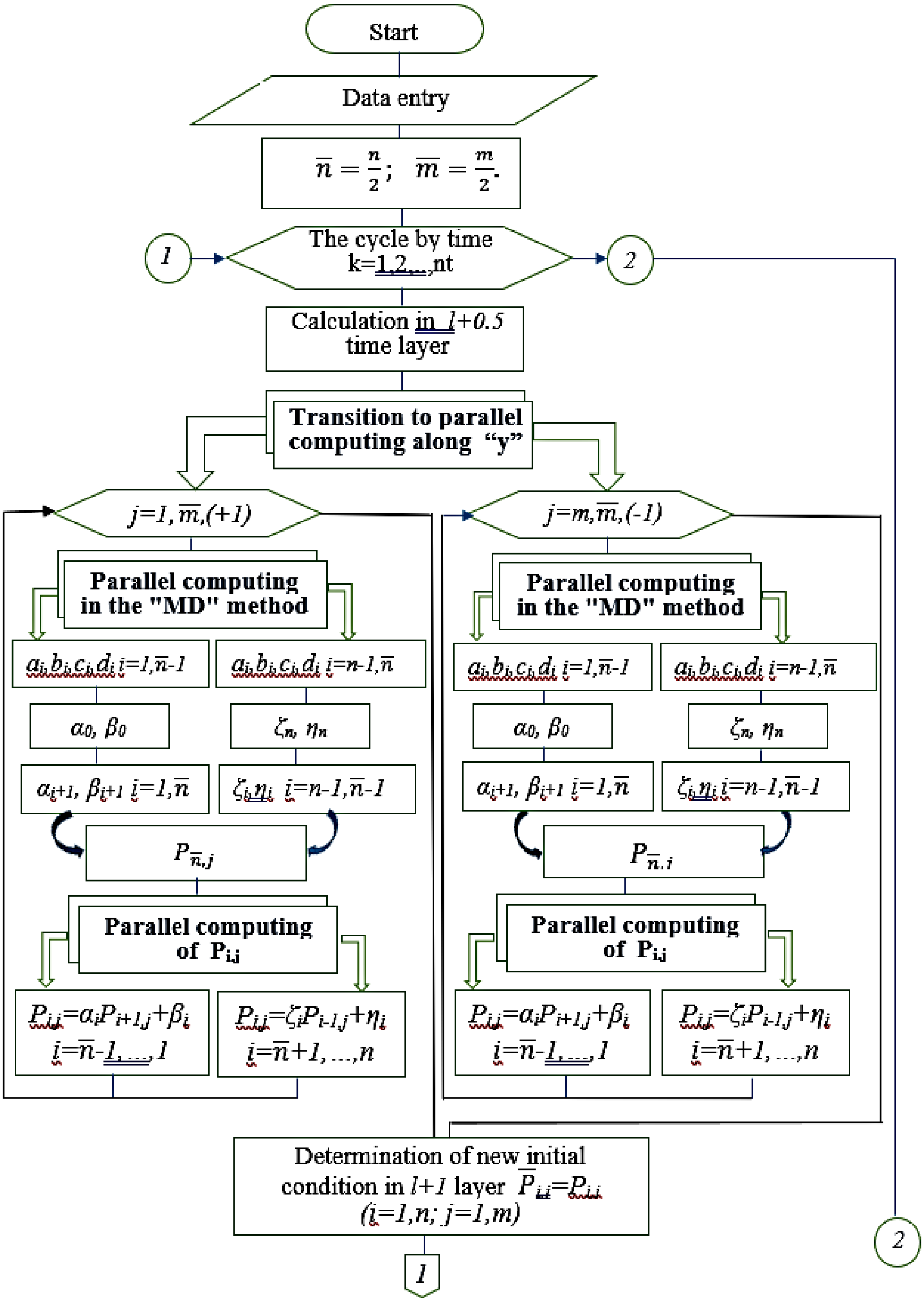

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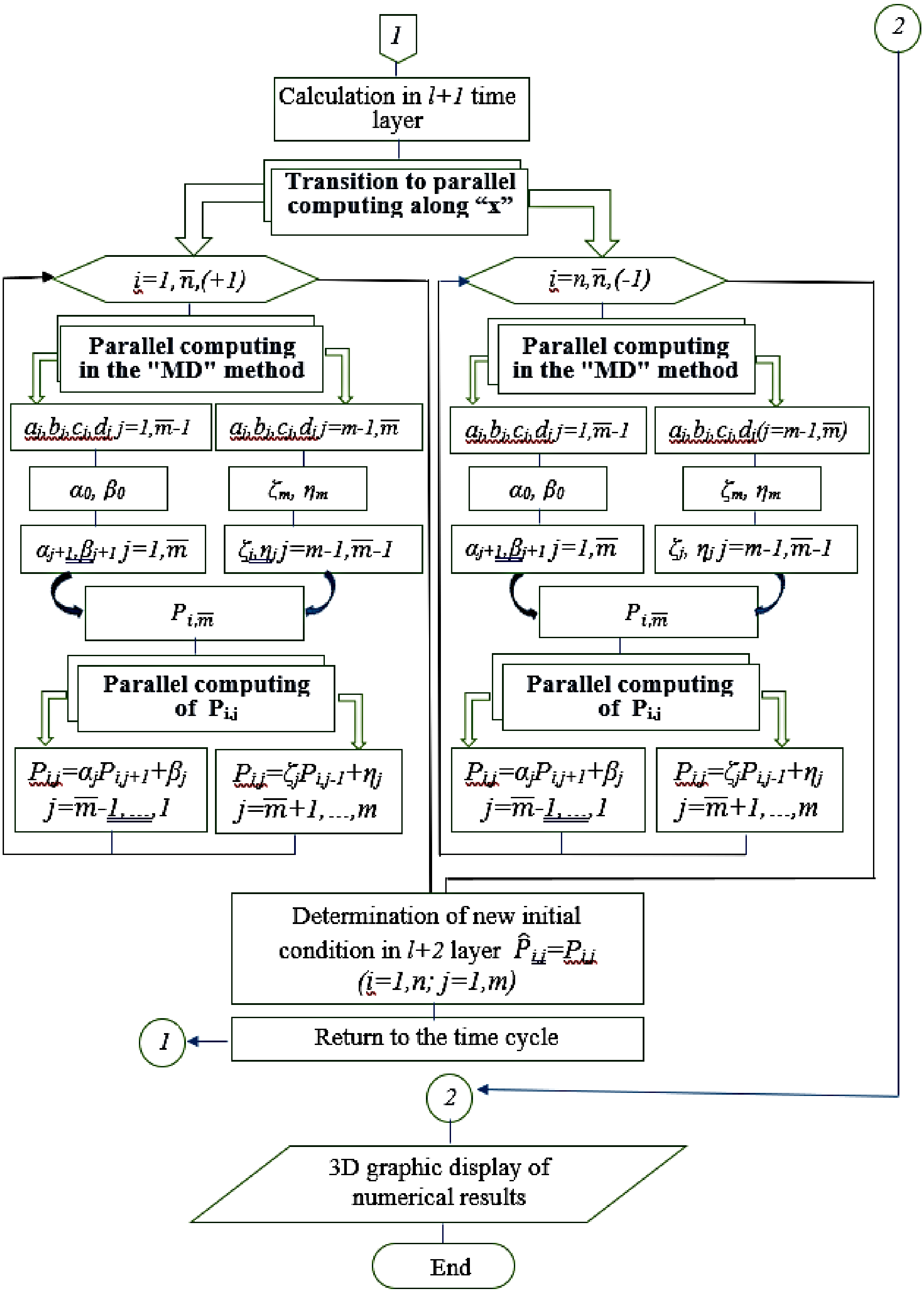

Figure 1. Cross-section scheme is based on the efficient parallel computing algorithm block scheme

Computational experiments. In the field of mathematical modeling, a number of computational experiments were carried out on a personal computer for different values of water filtration parameters in a porous medium. To study and determine the main indicators of water at pressurly and conduct a calculation experiment on a personal computer, we use the following preliminary data:

h (x,y) - starting water pressure,100 m.;

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μ^* - coefficient of water supply, 0.2;
K (x, y) - filtration coefficient, 1;
m -the power of the water source Horizon,10 m;
q -well debit, 100 m^3/day.

The time it takes for groundwater to calculate the main indicators of the filtration process depends on the size of the discrete field of filtration, that is, the number of points of the X and y axes in it and the calculation time. If in a discrete field there is a large number of points in each direction, and it is necessary to calculate the calculation time, that is, several years of indicators of Water Well operation, then the program will take a huge amount of time for full operation. In this case, the reduction of the computing process is carried out mainly through the creation of parallel computing algorithms and software modules suitable for them, as well as their effective use.

The algorithm used three main cycles. The first k-cycle indicates a change in time, while the second i and third j cycles mean a repetition in the direction of the X and y Point variables. Therefore, computational experiments were carried out on the number of points of coordinate lines and the total calculation time. The number of points on the coordinate axes of the first and second computational experiments (50,50), (100,100), (150,150) and (200,200) the calculation process was carried out in 360 and 1080 days (2, 3rd pictures), the second experiment was carried out number of points on the coordinate axes at 360, 720, 1080, 1440 days (50,50) and (100,100) (2, 3rd pictures).

As can be seen from images 2, 3.the increase in the size of the discrete filtration area is due to the fact that the parallel computing algorithm increases much more efficiency than the sequential calculation algorithm. In the same increase in computing time, there is also an increase in efficiency.

As can be seen from the tables and graphs in the pictures, when the program is working using a parallel calculation algorithm (Figure 1), the efficiency is more than 45-50% of both computing experiments and ensures one and a half to two times faster operation of the program.

The change in the water pressure in the layer over a 3 – year period is presented in 3D graphics in visual form in Figure 4.5.6.

From the experience of calculation, we can say that with an increase in the calculation time and the number of points of coordinate lines, the efficiency of the program's operation increases.

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Counting period(720 day)			
Number of points	**Sequential computing time**	**Sequential computing time**	**Efficiency**
50	117	75	1,56
100	439	306	1,43
150	987	638	1,55
200	1761	1100	1,60

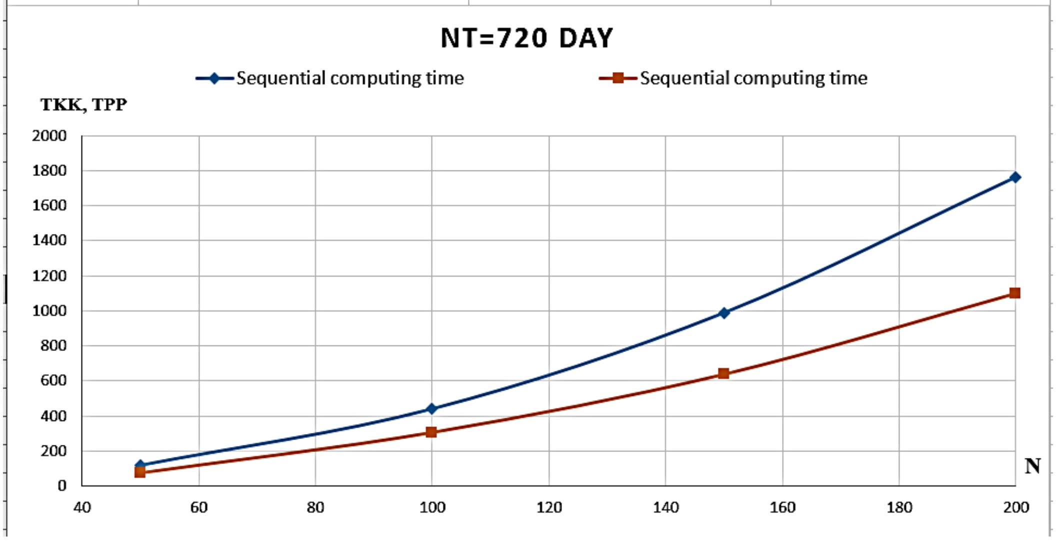

Figure 2. Results of work on the number of points of coordinate lines at a time of 720 days

Number of points(100)			
Counting period	Sequential computing time	Sequential computing time	Efficiency
360	226	160	1,41
720	441	307	1,44
1080	657	440	1,49
1440	873	573	1,52

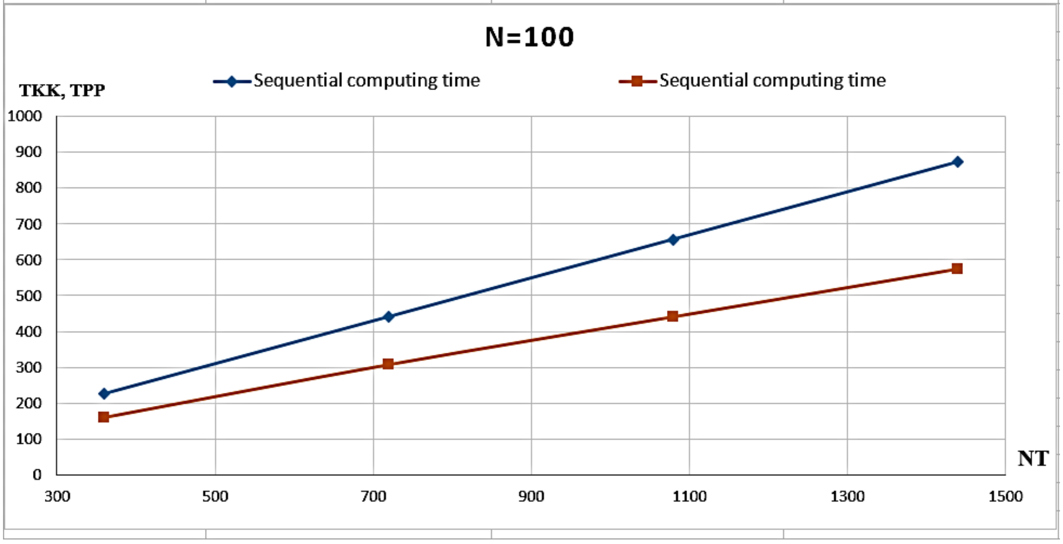

Figure 3. Calculation time results when the number of points is nx=100, ny=100

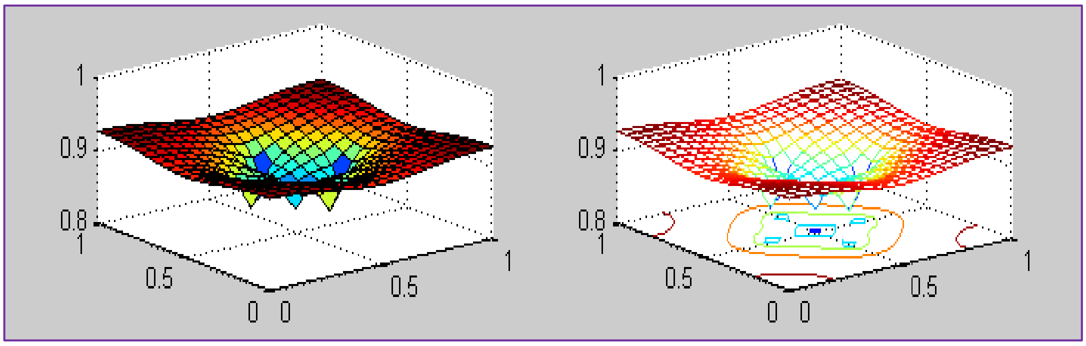

Figure 4. 3D graph of water pressure change in the layer

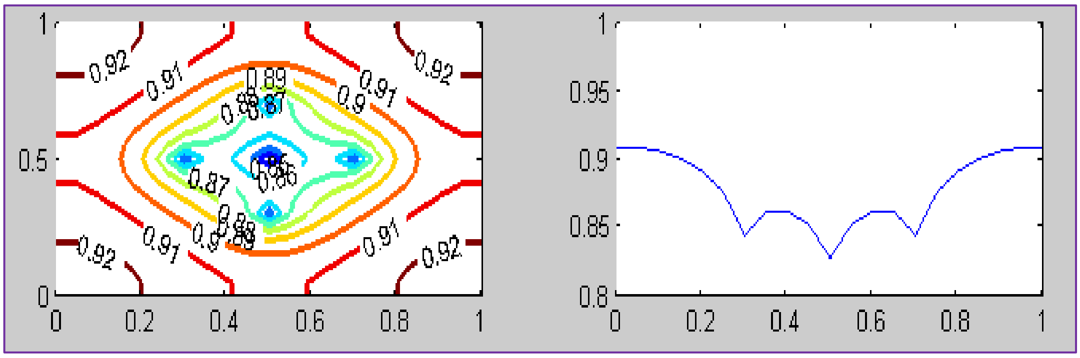

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Figure 5. Contour graph of water pressure change in layers and Wells

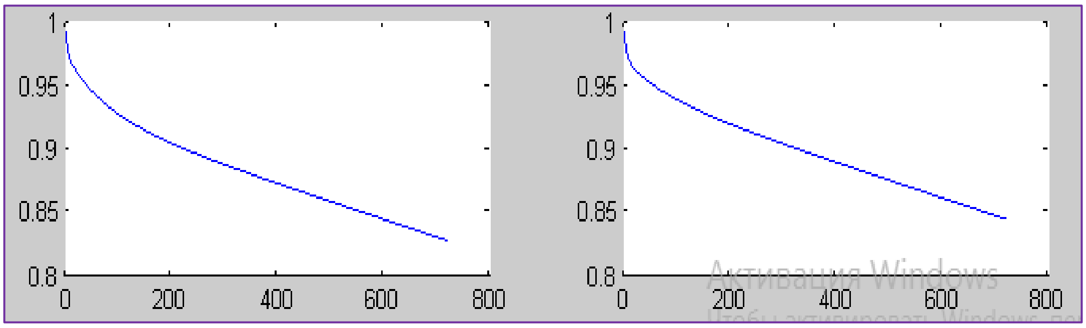

Figure 6. Graph of water pressure drop in Main and edge wells

Conclusion. This parallel computing algorithm significantly reduces the computing process on a computer when modern computers with two or four processors use special library modules of the C++ programming language OpenMP, Delphi, Matlab and other programming languages.

Thus, the developed parallel calculation algorithms can be widely used to solve two-dimensional equations of the parabolic equation type. For example, they are useful for solving two-and three-dimensional issues of filtration of oil and gas layers in a porous medium.

The calculation algorithm is based on the application of the cross-sectional direction scheme method. Structured mathematical models and their implementation calculation, parallel calculation algorithms and software, serve as the basis for solving classes of problems of the water filtration process in porous environments.

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